Propagation of Uncertainties

The propagation of uncertainties is simply a method to determine the uncertainty in a value where the value is calculated from two or more measured values with known uncertainties.

$$W = Fs\cos\theta = f(F,s,\theta)$$

where we measured $F \pm \delta F$, $s \pm \delta s$, and $\theta \pm \delta\theta$
and we want to calculate $W \pm \delta W$

$$E_k = \frac{1}{2}mv^2 = f(m,v)$$

where we measured $m \pm \delta m$ and $v \pm \delta v$
and we want to calculate $E_k \pm \delta E_k$

$$E_{p,g} = ma_g y = f(m,y)$$

where we measured $m \pm \delta m$ and $y \pm \delta y$
and we want to calculate $E_{p,g} \pm \delta E_{p,g}$

In general, we have some value, w, that we want to calculate that is a known function of the measured values, x, y, and z.

$$w = f(x,y,z)$$

where we measured $x \pm \delta x$, $y \pm \delta y$, and $z \pm \delta z$
and we want to calculate $w \pm \delta w$

The basic formula to find the uncertainty, δw, is given by:

$$\delta w = \sqrt{\left(\frac{\partial w}{\partial x}\delta x\right)^2 + \left(\frac{\partial w}{\partial y}\delta y\right)^2 + \left(\frac{\partial w}{\partial z}\delta z\right)^2} \qquad (1)$$

Two of the more common situations we encounter are

1: addition and subtraction of measurements

2: multiplication and division of measurements

Case 1: Addition and Subtraction of Measurements

Suppose that

$$w = ax + by + cz$$

where a, b, and c are known positive or negative constants, and x, y, and z are measured values with known uncertainties δx, δy, δz.

Then the uncertainty in w from equation (1) is given as

$$\delta w = \sqrt{(a\delta x)^2 + (b\delta y)^2 + (c\delta z)^2}$$

Case 2: Multiplication and Division of Measurements

Suppose that

$$w = k\,x^a y^b z^c$$

where k, a, b, and c are known positive or negative constants, and x, y, and z are measured values with known estimated uncertainties δx, δy, δz.

Then the uncertainty in w from equation (1) is given as

$$\delta w = w\sqrt{\left(\frac{a\,\delta x}{x}\right)^2 + \left(\frac{b\,\delta y}{y}\right)^2 + \left(\frac{c\,\delta z}{z}\right)^2}$$