Unit 13.1

- Playing a record? I'll show you something interesting.

- Compare a point on the label with a point on the record's outer edge. They both make a complete circle in the same amount of time, right?

  Yeah...

- But the point on the record's edge has to make a bigger circle in the same time, so it goes faster. See, two points on one disk move at two speeds, even though they both make the same revolutions per minute!
Unit 13.1

For this unit, we will

– refine our definition of \( \tau \equiv r F \).
– develop the vector nature of rotational quantities \( \theta, \omega, \alpha, \tau \).
– introduce the cross (vector) product \( \vec{A} \times \vec{B} \)
– define rotational momentum, \( \vec{L} \) (angular momentum)
– develop rotational Impulse–Momentum Theorem
  \( \Rightarrow \) Conservation of Rotational Momentum.

In developing these new concepts, I’m going to ask you to know and use a lot of concepts from earlier units, and from Unit 12 in particular.
Unit 13.2

Recall from Unit 10 when we developed the concept of **Work**.

First definition (when $\vec{F}$ and $\Delta \vec{r}$ were parallel to each other):

$$ W \equiv F_x \Delta x $$

Refined definition:

$$ W \equiv \vec{F} \cdot \Delta \vec{s} = |\vec{F}| |\Delta \vec{s}| \cos \theta $$

[dot (scalar) product]
We want to do the same thing for the concept of **Torque**.

First definition (when $\vec{F}$ and $\vec{r}$ were perpendicular to each other):

$$|\vec{\tau}| = |\vec{r}| |\vec{F}|$$

Refined definition:

$$\vec{\tau} \equiv ? \text{ (magnitude?, direction?)}$$

How can we **unambiguously** define direction of rotational quantities?
Unit 13.3

Explore the concept of the cross (vector) product and the Right Hand Rule (RHR).

**Angle Method**

\[ \vec{\tau} = |\vec{r}| |\vec{F}| \sin \theta \quad + \quad \text{RHR} \]

(magnitude) + (direction)

**Formal Method**

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

\[ = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \]

\[ = (r_y F_z - r_z F_y) \hat{x} - (r_x F_z - r_z F_x) \hat{y} + (r_x F_y - r_y F_x) \hat{z} \]
## Unit 13.4

Translational quantities:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Translational</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>$\bar{x}, \bar{y}, \bar{z}$</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>$\bar{v}$</td>
<td>$\frac{d\bar{r}}{dt}$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$\bar{a}$</td>
<td>$\frac{d\bar{v}}{dt}$</td>
</tr>
<tr>
<td>External Interaction</td>
<td>$\bar{F}$</td>
<td></td>
</tr>
<tr>
<td>Inertia</td>
<td>$m$</td>
<td></td>
</tr>
<tr>
<td>Second Law</td>
<td>$\bar{a} = \frac{\bar{F}^{net}}{m}$</td>
<td></td>
</tr>
</tbody>
</table>
Rotational analogs:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Rotational</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>$\vec{r}, \vec{\theta}, \vec{z}$</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>$\vec{\omega}$</td>
<td>$\frac{d\vec{\theta}}{dt}$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$\vec{\alpha}$</td>
<td>$\frac{d\vec{\omega}}{dt}$</td>
</tr>
<tr>
<td>External Interaction</td>
<td>$\vec{\tau}$</td>
<td>$\vec{r} \times \vec{F}$</td>
</tr>
<tr>
<td>Inertia</td>
<td>$I$</td>
<td>$\int r^2 dm \ (= km d^2)$</td>
</tr>
<tr>
<td>Second Law</td>
<td>$\vec{\alpha} = \frac{\vec{\tau}^{\text{net}}}{I}$</td>
<td></td>
</tr>
</tbody>
</table>
Two other rotational quantities that we use:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Translational</th>
<th>Rotational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum</td>
<td>$\vec{p} \equiv m\vec{v}$</td>
<td>$\vec{L} \equiv \vec{r} \times \vec{p}$ [\equiv I\vec{\omega}]</td>
</tr>
<tr>
<td>Kinetic Energy</td>
<td>$K_{\text{tran}} \equiv \frac{1}{2}mv^2$</td>
<td>$K_{\text{rot}} \equiv \frac{1}{2}I\omega^2$</td>
</tr>
</tbody>
</table>
Unit 13.5

A couple of things to really think about for this section:

– Don’t confuse “the torque needed” with “the force needed”.

– In translational motion, $\vec{v}$ and $\vec{p}$ point in the same direction.
  $\Rightarrow$ $\vec{\omega}$ and $\vec{L}$ point in the same direction.

– In translational motion, $\vec{F}^{net}$ and $\Delta\vec{p}$ are always in the same direction, but $\vec{F}_{\text{net}}$ and $\vec{p}$ won’t in general be in the same direction.
  $\Rightarrow$ The same will be true for $\vec{\tau}^{net}$, $\Delta\vec{L}$, and $\vec{L}$.

This will definitely stretch our brains today and next class.

Draw Pictures!
Unit 13.6

Rotational First and Second Law of Motion.

To help us by analogy, let’s remind ourselves what we did for the Translational First and Second Law of Motion.

Newton’s First Law of Motion (law of inertia).
- If the net force on an object is zero, then the velocity is zero or constant. (If $\vec{F}_{\text{net}} = 0$, then $\vec{v} = 0$ or constant)
- If the net force on an object is zero, then the change in momentum is zero. (If $\vec{F}_{\text{net}} = 0$, then $\Delta \vec{p} = 0$)

Newton’s Second Law of Motion.
- The acceleration of an object is directly related to the net force and inversely related to the translational inertia. ($\ddot{a} = \frac{\vec{F}_{\text{net}}}{m}$)
- The time rate of change of momentum of an object is directly related to the net force. ($\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}$)