Sample Project Report

This sample is based roughly on a report written by the authors in 1992

Using Video Analysis to Verify Coulomb's Law of Electrostatics

by

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Purpose

The purpose of this experiment is to test Coulomb's Law which states that the force between two spherically symmetric charged objects is directly proportional to the product of the charges, and inversely proportional to the square of the distance between the centers of the two charges. In mathematical vector notation Coulomb's Law is expressed as

$$\vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

where $\vec{F}_{12}$ is the force on particle 1 due to particle 2 in Newtons, q_1 is the charge on particle 1 in Coulombs, q_2 is the charge on particle 2 in Coulombs, $\hat{r}_{12}$ is a unit vector originating at the center of particle 1 and pointing directly away from particle 2, r is the distance between the centers of the two particles in meters, and k is a constant, given by

$$k= 8.99 \text{ X } 10^9 \ [\text{N}\bullet\text{m}^2/\text{C}^2].$$

Introduction

Coulomb formulated his law and tested it using a device he called the torsion balance. One sphere suspeded from a fiber has the same charge as a second sphere. These spheres are charged in the same manner so they repel each other. This causes the the fiber supporting one of the spheres to twist. To measure this repulsive force, Coulomb counter-acted the repulsive force by twisting the suspension head through the angle theta needed to keep the two spheres a certain distance apart. The force of repulsion was balanced by the force resulting from the twisting of the suspension head. Thus, the angle theta provided a relative measure of the force of repulsion between the two spheres. Coulomb performed a similar experiment to test the force of attraction between two spheres of opposite charges.

We used the video analysis of a different set up to investigate this law. Our setup consisted of a charged ball suspended by two strings, and a charged ball mounted on a lucite prod. This is shown in Fig. 1. The prod has the same charge as the suspended sphere. Since like charges repel, the ball moves away from the prod as it is brought closer. The distance that the suspended ball moves can be used as a measure of the force of repulsion.

Equations

To calculate the repulsive force using our method, a look at the forces acting on the sphere and how they are related mathematically is helpful. In Figure 1, the charged ball has been displaced from equilibrium by a force, $\vec{F}_c = \vec{F}_{12}$. This force is due the prod with a charge

q_2 on it which is placed on the same horizontal line as q_1. If q_1 is at rest then it is in equilibrium, and thus the net force on it,

$$\vec{F}_{net} = \sum \vec{F} = 0$$

Using $\hat{x}$, $\hat{y}$ notation for these vectors, the tension on the string, $\vec{F}_T$, the force of gravity, $\vec{F}_g$, and the force due to the prod, $\vec{F}_c$ can be defined as follows:

$$\vec{F}_g = (|\vec{F}_g| \sin \theta)\,\hat{x} + (|\vec{F}_g| \cos \theta)\;\hat{y}$$

$$\vec{F}_g = 0\,\hat{x} - |\vec{F}_g|\;\hat{y}$$

$$\vec{F}_c = -|\vec{F}_c|\,\hat{x} + 0\,\hat{y}$$

Adding these vectors gives

$$\sum \vec{F} = (|\vec{F}_g|\; \sin \theta - |Fc|)\hat{x} + (|FT| \cos \theta - |Fg|)\;\hat{y} = 0$$

The zero vector is defined as $0\,\hat{x} + 0\;\hat{y}$, and by the properties of vector equivalence, two equal vectors must have equivalent components, so

$$|\vec{F}_g| \sin \theta - |Fc| = 0 \qquad \text{(eq. 1)}$$

$$|\vec{F}_g| \cos \theta - |Fg| = 0 \qquad \text{(eq. 2)}$$

Solving eq. 2 for $|\vec{F}_g|$ and using the fact that $|Fg| = ma_g$, where m is the mass of the ball and a_g is the acceleration of gravity,

$$|\vec{F}_T| = \frac{ma_g}{\cos\theta}\,|$$

Substituting this result into eq. 1, and solving for $|\vec{F}_c|$

$$|\vec{F}_c| = \frac{ma_g \sin\theta}{\cos\theta}$$

which can be rewritten using the definition of the tangent function as

$$|\vec{F}_c| = ma_g \tan\theta \qquad \text{(eq. 3)}$$

In Figure 1, we see that

$$\tan\theta = x/y \qquad \text{(eq. 4)}$$

$$x^2 + y^2 = L^2 \qquad \text{(eq. 5)}$$

Equation 4 is the definition of tangent, and equation 5 is the Pythagorean Theorem. The effective length, L, is the distance from the ball to the point of suspension and not the length of the string, which can be seen in Figure 2. Solving eq. 5 for y and substituting this result into eq. 4,

$$\tan\theta = x/\sqrt{L^2 - x^2}$$

Using this result equation 3 can be written as

$$|\vec{F}_c| = ma_g x/\sqrt{L^2 - x^2} \qquad \text{(eq. 6)}$$

Methods and Materials

In this investigation two ping-pong balls that were covered with conducting paint were stroked with a fur-charged rubber rod and touched together to equalize their charges. One of the negatively-charged balls is hung from a long bifilar pendulum and the other, which serves as a prod, was attached to an insulated rod, as shown in Figure 1.

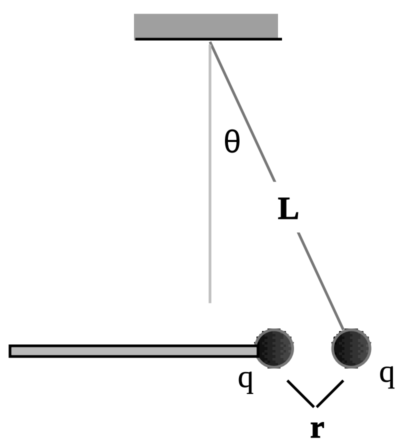

Figure 1: A charged ping-pong ball is repelled from an equally charged prod. At equilibrium, the vector sum of the gravitational force, the tension in the string, and the Coulomb force on the hanging ball is zero.

The hanging ball is pushed to larger angles and rises higher as the prod is brought closer to it. This experiment demonstrates qualitatively that the force exerted by the prod on the hanging ball is greater when the distance, r, is smaller between the centers of the two charged balls. However, new technology allows us to take this experiment a step further. The slow motion of the prod inching forward can be captured with a video camera and digitized. Then the VideoPoint™ software can be used to perform a frame-by-frame analysis of the angular displacement of the hanging ball and of the distance between the balls. This yields the information needed to find the magnitude of the Coulomb force, F_C, as a function of r. Figure 2 shows the set-up.

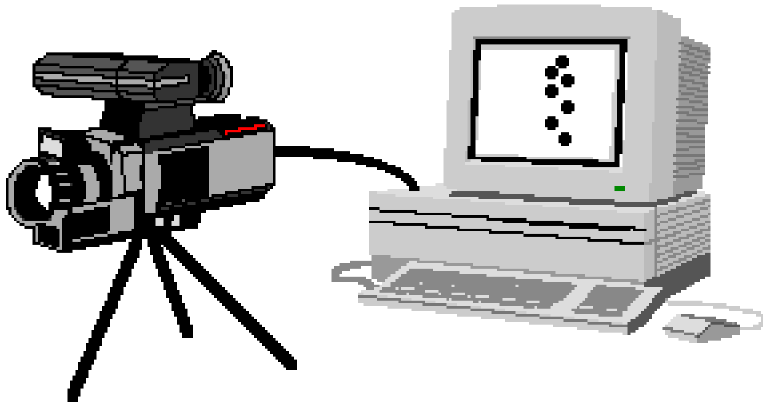

Figure 2: A video camera is attached to the video input of a Macintosh 7200 computer so that digital images of the video frames showing the motion of the hanging ball can be recorded for analysis by the VideoPoint software.

Twenty five frames showing the position of the hanging ball as the prod was brought closer and closer to it are shown in Figure 3.

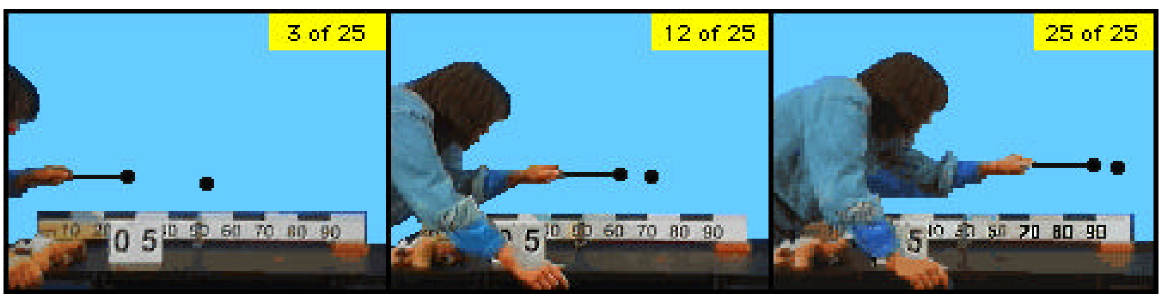

Figure 3: Three of twenty-five digitized video frames depicting the forces between two charged balls. The string holding up the hanging ball is not visible.

Data Collection:

In using the VideoPoint program to analyze the digital video images, we started by renaming Point Series 1 as "hanging ball". Next the mouse was used to place a marker over the center of the hanging ball in each frame and then clicked to allow the program to record the "hanging ball" location in each frame. Next a Point Series was created called "prod" and the center of the prod was located in each frame. The x- and y- coordinates of the hanging ball and rod were recorded in the VideoPoint data table.

Along with this table, we recorded the equilibrium position of the ball, the mass of the ball, the mass per meter of the string, and the effective length of the string. The total length was found by measuring the distance between the two points of suspension and using the Pythagorean theorem. These values are shown above the spreadsheet. Locations of several points on one of the digitized frame that were 10 cm apart along the image of meter stick were recorded to scale the movie to obtain prod and ball the coordinates in meters instead of pixels.

Calculations

To determine if F_c is inversely proportional to r^2, we calculated Fc and 1/r2. Several other calculations were made before we were able to arrive at these values. In table 1, columns 1 and 2 are data. Columns 3 and 4 are the coordinates of the ball and prod in meters. The conversion was made using the following equation.

$$x_m = (x_p - 109.2)/.1112$$

where xm is the coordinate in meters, and xp is the coordinate in pixels. This equation was found by graphing pixels vs. meters for known values along a meter stick. The graph was linear with an R^2 value of 1.000, and the equation of the least squares fit line was used to convert from pixels to meters.

The distance between the prod and the ball, r, is the difference in the two coordinates;

$$r = x_p - x_b$$

where x_p is the x coordinate of the prod and xb is the x coordinate of the ball. Column 7, 1/r2 was computed from r. The distance from equilibrium, x, was calculated using

$$x = x_e - x_b$$

where x_e is the coordinate of equilibrium. The magnitude of the repulsive force, F_c, is found using the equation derived earlier, adding the mass of the string to the mass of the ball.

Results:

If F_c is inversely proportional to r^2, then a graph of F_c vs. r should yield an inverse square relationship. A data summary is shown in Table 1 below.

Table 1: Summary of Coulomb Data

x-ball	r(m)	F data	F model
-0.002	0.349	-1.8E-05	9.49E-05
-0.005	0.282	-5.5E-05	1.46E-04
-0.002	0.220	-1.8E-05	2.40E-04
0.022	0.210	2.2E-04	2.62E-04
0.005	0.177	5.5E-05	3.68E-04
0.026	0.176	2.6E-04	3.76E-04
0.031	0.165	3.1E-04	4.28E-04
0.031	0.146	3.1E-04	5.41E-04
0.060	0.143	6.1E-04	5.69E-04
0.059	0.119	5.9E-04	8.20E-04
0.088	0.124	8.9E-04	7.49E-04
0.095	0.104	9.6E-04	1.07E-03
0.121	0.101	1.2E-03	1.15E-03
0.143	0.091	1.4E-03	1.38E-03
0.163	0.082	1.6E-03	1.71E-03
0.196	0.082	2.0E-03	1.71E-03
0.208	0.077	2.1E-03	1.95E-03
0.223	0.075	2.3E-03	2.06E-03
0.254	0.070	2.6E-03	2.38E-03
0.263	0.064	2.7E-03	2.82E-03
0.267	0.064	2.7E-03	2.82E-03
0.280	0.059	2.8E-03	3.37E-03
0.313	0.066	3.2E-03	2.66E-03

As shown in Figure 4, when the video analysis is done carefully and modeled with $F_c = \text{Const.}/r^2$, the inverse square relationship is quite good.

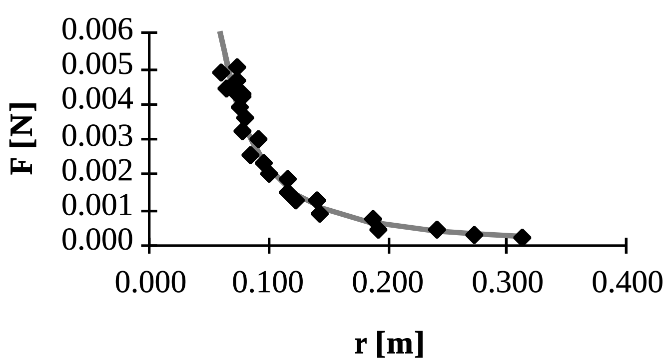

Figure 4: A graph of the Coulomb Force vs. the distance between two charged ping-pong balls. The gray line is based on a fit of $F_c = \text{Const.}/r^2$ that shows that the inverse square law holds quite well. Assuming that Coulomb's Law holds, it can also be shown that each ball carries about 5×10^{-8} Coulombs of excess charge. The VideoPoint® software was used to obtain the data from video frames.

In the graph above the last five points were not considered. This is because these last five points represent a time when the two charged spheres were at very close range. The distance separating the two spheres is close to one diameter. This close range caused the repulsive force between the two spheres, F_C, to distort the charge distribution. The graph curves down because not all of the force is going into displacing the ball, so that the experimental F_C calculated from the work done against gravity is small than actual F_C. F_C is smaller than expected because the distance between the centers of the charge distributions is now larger than the distance between the centers of the spheres. The distortion of charge, called induction, results in the spheres have a larger charge density on their far surfaces instead of having a uniform charge density as they did at farther distances.

Conclusions:

This experiment shows that Coulomb's Law holds to the limits of uncertainty in our experiment. The spheres in our experiment were not point charges and so at close distances they experienced charge by induction due to the other sphere. This caused the force of repulsion to appear small than expected at close range because the effective distance between the charges was larger than the distance r, by a factor of almost 1 cm, the diameter of the spheres.

Another source of uncertainty in our experiment is that the charge on a sphere will eventually leak into the atmosphere. Water is the main agent in this transfer of charge. Thus, on a cold, dry day the effect is small. We preformed our experiment on a bitterly cold night and this did not affect our results as much as it could have on a damp day.