Examination of the Magnetic Field Directly Outside the Edge of a Solenoid in Relation to the Theoretical Magnetic Field Inside A Solenoid

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Purpose

The goal of this project is to examine how the predicted magnetic field inside a solenoid (a conducting wire wound in a tight helical coil of many turns) compares to the magnetic field measured directly outside the ends of a solenoid so we can formulate a relationship between the two different field measurements. In order to find the predicted magnetic field, we will use the equation is expressed in the form:

$$B = \mu_o n I_o$$

where β is the magnetic field inside the solenoid in Teslas, $I_o$is the current in one wire in Amps, *n* is the ration *N/l,* the number of turns of wire per unit length of the solenoid in inverse meters, and μ_o is the permeability constant given by: [3]

$$\mu_o = 1.26 \times 10^{-6} \frac{N \cdot m^2}{C^2}$$

Therefore, our project will primarily focus on altering the current through the solenoid and the number of turns per wire to see what effect these variables have on the magnetic field outside the solenoid and how they compare to what would happen if these same changes took place but the magnetic field was being measured inside the solenoid. If our data matches our predictions, then as the number of loops per meter increases B will increase, and as the current increases B will increase.

*Note, although the equation is for an infinitely long solenoid as long as the solenoids length is large compared to its width the equation still holds.

Relevant Theory and Equations

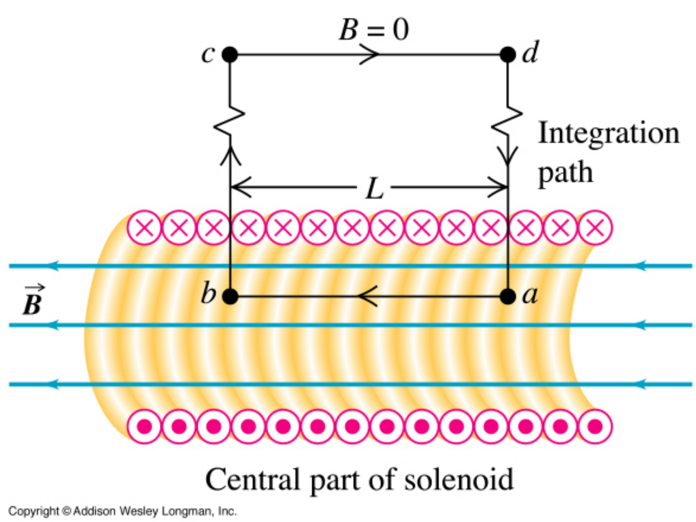
Hans Christian Oersted observed in the early 19th Century that when an electric current was passed through wire that it generated a magnetic field. Encouraged by the discoveries of

Oersted, Andre-Marie Ampere investigated the relationship between currents and magnetic fields and formulated Ampere's Law:

The integral around a closed path of the component of the magnetic field tangent to the direction of the path equals μ_o *times the current intercepted by the area within the path:* [3]

$$\oint B_{\parallel}\, ds = \mu_o I \qquad \text{(eq. 1)}$$

where the circle on the integral means that the integration of $B_{\parallel}$ is performed around a closed path, μ_o is the permeability constant, and *I* is the current intercepted by the area of the closed path. See figure 1.



(Fig. 1- Application of Ampere's Law to a Solenoid) [1]

Note that in this picture the solenoid is tightly wound and symmetric which is important since Ampere's Law requires that the distribution of the currents be highly symmetrical. The right hand rule for solenoids can be used to explain the direction of the magnetic field lines:

Right-Hand Rule for Solenoids: If the fingers of the right hand curl around the solenoid in the direction of the current, the thumb gives the direction of the magnetic field inside the solenoid.

Using figure 1 as an example one can determine the magnetic field inside the solenoid using Ampere's Law. Since the paths *ad* and *bc* are tangent to the direction of the magnetic field they result in a magnetic field component of zero. Also, since the path *cd* falls outside the solenoid this side of the square has a magnetic field component of zero. However the magnetic field has a component tangent to a path that is inside the solenoid, this path being *ab*. Since the field is completely symmetric throughout the solenoid then the magnitude of $B_{\parallel}$, or in this case since it is already parallel, *B*, is constant along this side of the square. Therefore using Ampere's Law:

$$\oint B_{\parallel}\, ds = 0 + 0 + 0 + \int_0^l B_{\parallel}\, ds = Bl = \mu_o I \qquad \text{(eq. 2)}$$

However, the current intercepted by the area of the square in this case is $I_o \times N$, where I_o is the current in one wire and *N* is the number of wires intercepted by the area of the square.

Therefore:

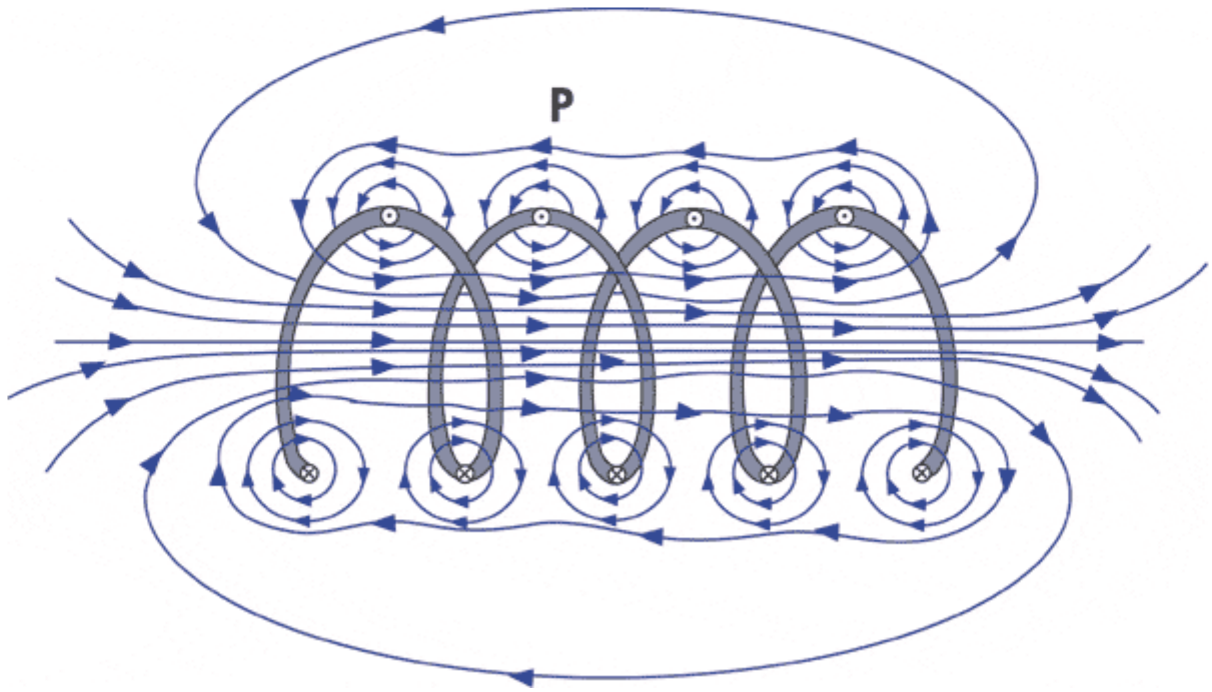
$$Bl = \mu_o I_o N$$

From which

$$B = \mu_o I_o \frac{N}{l} = \mu_o I_o n \qquad \text{(eq. 3)}$$

Since, as defined earlier, *n* is the common term for the ratio of the number of turns of wire per unit length of the solenoid, *N/l*. [3]

Lastly, since this experiment measured the magnetic field directly outside the ends of solenoids, it should be illustrated what effect this might have on the magnetic field. See figure 2.



(Fig. 2- Magnetic Field Inside and Directly Outside a Current-Carrying Solenoid)[2]

Since magnetic field lines have no beginning or end this results in the loop pattern seen in the diagram. Therefore, as depicted in figure 2, and expected since the magnetic field is constant throughout the solenoid, the magnetic field lines don't start looping until they reach the end of the solenoids effectively decreasing the line density at these points since they spread out from each other. Lastly, since a lower magnetic field line density means a weaker magnetic field this diagram supports that the magnitude of the magnetic field should be weaker just outside the edge of the solenoid compared to the magnitude inside the solenoid. It is this relationship of how much this magnitude differs from the value calculated using Ampere's law that we are interested in studying.

Methods and Materials:

In order to investigate the magnetic field outside a solenoid, solenoids had to be created. To this end, insulated wire was wrapped around circular objects of varying radius and material. Once the wire had been looped the desired amount of times around the object, the solenoid was slid off of it, except in the case of three solenoids where they remained wrapped around their respective objects were held in place by electrical tape and rubber bands. The wire was ridged enough that it maintained its shape after being slid off whatever it was wrapped around. The wires were then stripped of their insulation at both ends with a knife so that it could be connected to a circuit.

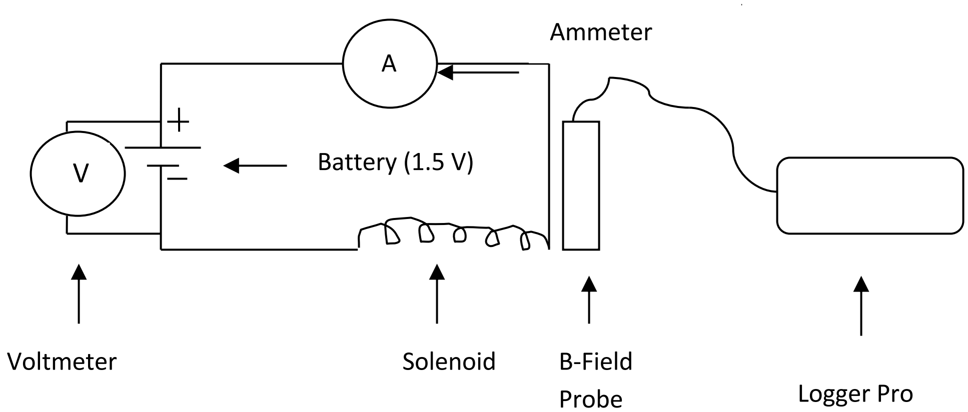
The circuit was created by connecting a 1.5V battery or multiple 1.5 V batteries to either side of the solenoid with alligator clips. The battery's voltage and the current of the circuit were tested after each use of it by a multi-meter and an ammeter respectively. Once a circuit was closed, a magnetic field probe was used to measure the strength of the field while Logger Pro was used to record it.

In experiment 1, we compared the magnetic field created by solenoids of varying numbers of loops and lengths. Eight different solenoids were created and used. Although neither the length of the solenoids nor the number of loops they had was kept constant, we were looking at the ratio of loops per meter of wire (N/l). The current, wire used, and voltage were kept constant.

In experiment 2, we compared the magnetic field created by a solenoid with varying currents running through it. The same solenoid was used and four trials were run, each using a different current.

Data Collection:

Data was collected using Logger Pro and a magnetic field probe. We found that the probe recorded the field differently depending on how it was oriented, so we turned the probe until we found which direction it measured the highest magnetic field at and kept its orientation the same for every trial. When the solenoid was placed in a circuit with the battery, the end connected to the positive end of the battery was held up to the probe so that the end of the solenoid was directly touching the probe. Data was then collected of the field measured in that area. Then the solenoid was removed so that data could be collected of the field that was absent of the solenoid. That way if the magnetic field of the space the probe was occupying for one measurement happened to be generally higher than the space it occupied during a different reading, the difference could be found between the two in order to minimize any discrepancies. The apparatus setup is displayed in figure 3:



(Fig. 3- Experiment Setup)

Data:

As stated in the material and methods, since the copper and plastic, as well as the square symmetric solenoid, had no noticeable change in their magnetic field due to these variations, and were completely dependent only on their *n* value, the solenoids were all classified with respect to their *n* value. This should be expected since the theory doesn't include these variables.

In table 1 the results concerning the data we collected to determine what affect changing the number of turns per unit length will affect the magnitude of the magnetic field directly outside the solenoid are displayed. The solenoids, A-H, are listed in order of increasing *n* value, and the percent difference in the last column is the percent difference between the theoretical magnetic field inside the solenoid value and the experimental magnetic field value directly outside the solenoid that we measured.

Solenoid	N	Length (m)	*n*=N/l (1/m)	Theoretical *B* (T)	Experimental *B* (T)	Experimental Uncertainty (+/- T)	Percent Difference
A	20	0.025	800	2.8E-03	2.44E-04	2.0E-05	91.3
B	42	0.04	1050	3.7E-03	2.86E-04	2.9E-05	92.3
C	122	0.11	1109	3.9E-03	2.93E-04	2.5E-05	92.5
D	50	0.041	1219	4.3E-03	4.04E-04	1.3E-05	90.6
E	50	0.041	1219	4.3E-03	4.20E-04	2.2E-05	90.2
F	37	0.03	1233	4.4E-03	3.94E-04	3.3E-05	91.0
G	74	0.06	1233	4.4E-03	4.07E-04	1.0E-05	90.8
H	28	0.02	1400	4.9E-03	4.92E-04	3.5E-05	90.0

Table 1: Comparison of Theoretical Magnetic Field vs. Experimental Magnetic Field Dependent on *n* with A Constant Current of 2.8 A

In table 2, the results concerning the data we collected for how changing the current and keeping the solenoid's *n* value constant affects the magnitude of the magnetic field directly outside the solenoid are displayed. The solenoid is the same in every measurement here, the values 1-4 are used to display the solenoid's measurements in order of increasing current. The percent difference in the last column has the same meaning of that in table 1.

Solenoid	Current	Theoretical *B* (T)	Experimental *B* (T)	Experimental Uncertainty (+/- T)	Percent Difference
1	2.8	4.40E-03	3.94E-04	3.30E-05	91.0
2	3.8	5.90E-03	5.31E-04	2.30E-05	91.0
3	4.5	7.00E-03	5.94E-04	2.10E-05	91.5
4	5.1	7.90E-03	7.29E-04	4.40E-05	90.8

Table 2: Comparison of Theoretical Magnetic Field vs. Experimental Magnetic Field Dependent on I_o with a constant *n* of 1233 m^{-1}

Calculations and Data Analysis:

To calculate the *n* value for each solenoid we took the *N* value, which was determined simply by counting the number of turns of wire on the solenoid, and divided it by the *l* value, which was simply found by measuring the solenoid from the first loop of one end all the way to the final loop at the other end of the solenoid. Therefore:

$$n = \frac{N}{l}$$

For example, Solenoid A that is .025m and has 20 loops,

$$n_A = \frac{20}{.025m} = 800m^{-1}$$

To determine the theoretical magnetic field inside a solenoid with the dimensions we had we used equation 3.

$$B = \mu_o I_o \frac{N}{l} = \mu_o I_o n$$

For example, Solenoid B with n=1050m^{-1} and $I_o = 2.8\ A$,

$$B_B = \left(1.26 \times 10^{-6} \frac{Nm^2}{C^2}\right)(1050m^{-1})(2.8A) = 3.7 \times 10^{-3}T$$

We calculated the experimental uncertainty by using an excel spreadsheet to calculate the average magnetic field value for a certain measurement as well as the standard deviation. Then we used these values to calculate the standard deviation of the mean for each solenoids experimental magnetic field directly outside the edge of the solenoid with the equation:

$$\sigma_{mean} = \frac{\sigma}{\sqrt{N}} \quad \text{(eq. 4)}$$

Where N is the number of data points taken and σ is the standard deviation. For example, Solenoid 1 with $\sigma = 7.4 \times 10^{-4}\ T$ and N=500,

$$\sigma_{mean} = \frac{7.4\times10^{-4}\ T}{\sqrt{500}} = 3.30 \times 10^{-5} T$$

Lastly, to find the percent difference this equation was used:

$$\text{Percent Difference} = \frac{|Theoretical\ Magnetic\ Field - Experimental\ Magnetic\ Field|}{Theoretical\ Magnetic\ Field} \times 100\% \quad \text{(eq.5)}$$

For example, Solenoid 4 with theoretical magnetic field value equal to 7.90E-03 T and experimental magnetic field value equal to 7.29E-04 T:

$$\text{Percent Difference} = \frac{|7.90\text{E}-03\ \text{T} - 7.29\text{E}-04\ \text{T}|}{7.90\text{E}-03\ \text{T}} \times 100\% = 90.8\%$$

*Note, the calculation are carried out the same way in both tables, the only difference is that table 2 varies the current while keeping *n* constant. There were no calculations needed to determine the current, it was found using an ammeter as described in the material and methods section.

Data Analysis:

Although the percent difference is large this was something we suspected. Since the theoretical equation is for the magnetic field inside a solenoid and the fact that the magnetic field lines decrease in density at the edges of the solenoid where we measured it comes with not too much surprise that this difference was observed. Also, the fact that every experimental magnetic field measurement is less than the theoretical calculation fits in with our predictions based off of magnetic field properties. More importantly, though, is that all the percent differences are roughly the same, that is, near 90% less than the theoretical magnetic field calculation. In fact, the range on these percent differences is only 2.5% (found by subtracting the largest percent difference from the smallest). Equally important, our data shows that the solenoids experimental magnetic fields directly outside the solenoid's edges increased as the n was increased and I_o kept constant or when the I_o was increased and the n was kept constant. This is the relationship that equation 3 predicts for the magnetic field inside a solenoid. Therefore, the fact that all the measurements were nearly the same percent less than their theoretical values, and that their measured magnetic fields increased as predicted by the equation gives support that the magnetic field equation for inside a solenoid can apply for directly outside the solenoid with a little adjusting. This will be discussed more in the conclusion.

*Note, although the experimental uncertainties may look like really small numbers, based on the magnitude of the actual measured magnetic field values as well as the number of data points the computer was able to record these are reasonable magnitudes for the experimental uncertainty in our project.

Results:

For the trials where the current was kept constant and n was changed we can use the equation for a linear function as well as equation 3 ($B = \mu_o I_o n$) to graph our data. The intercept is zero since a magnetic field of zero would result in both sides of the equation equaling zero. Since current is held constant in table 1 the slope would be the constant $\mu_o I_o$ and the n would correspond to x since it is being changed in each solenoid. Therefore

$$B = (\mu_o I_o) * n + 0 \qquad \text{(eq.6)}$$

For the trials where n was kept constant and the current was changing we can do the same linear function technique but this time we get:

$$B = (\mu_o n) I_o + 0 \qquad \text{(eq. 7)}$$

Using these equations we were able to graph the theoretical magnetic field inside a solenoid value for both a changing n and I_o and compare it to the experimental magnetic field value measured directly outside a solenoid for both variables. The results are shown in figures 4 and 5.

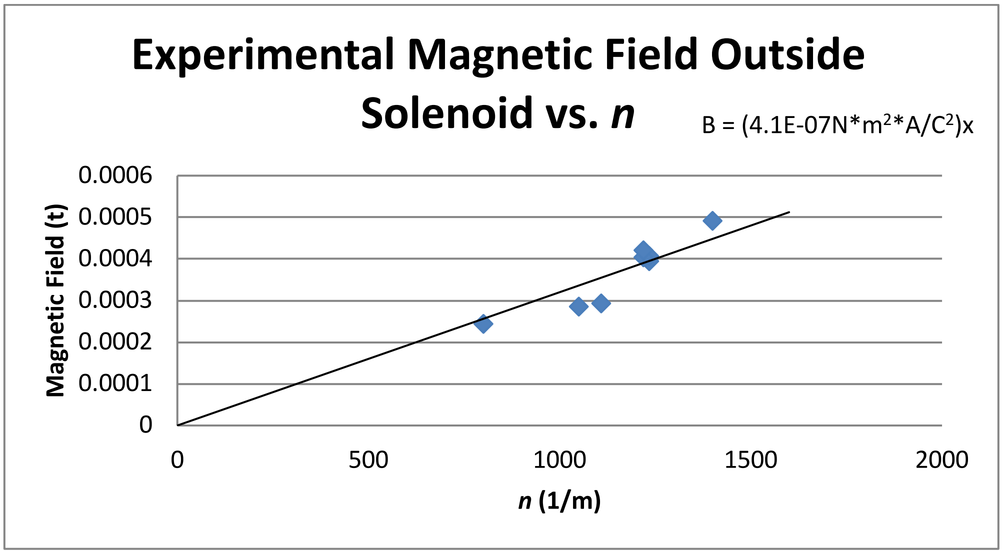


Figure 4: Experimental Magnetic Field where n is changing.

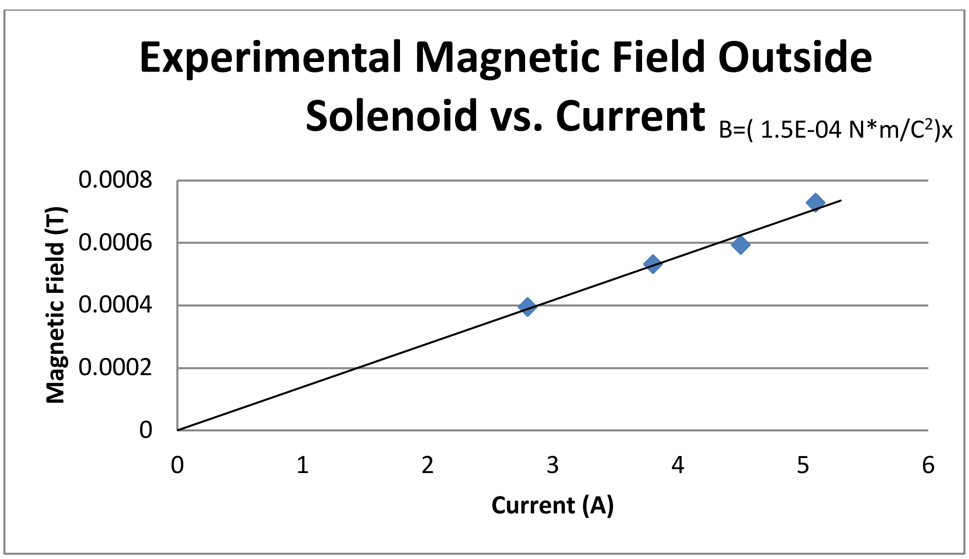


Figure 5: Experimental Magnetic Field where current is changing.

Using equation 6 we can calculate the experimental permeability constant based off the slope of the graph in figure 4. Since the slope of this graph is defined by $m = \mu_o I_o$ the experimental μ_o can be found by using the equation:

$$\mu_o = \frac{m}{I_o} = \frac{4.1 \cdot 10^{-7} \frac{N \cdot m^2 \cdot A}{C^2}}{2.8A} = 1.5 \cdot 10^{-7} \frac{N \cdot m^2}{C^2}$$

Using equation 7 and the equation displayed in figure 5 we can apply a similar technique to calculate the experimental permeability constant for this set of data. Since slope of the graph in figure 5 is defined by $m = \mu_o n$ the experimental μ_o in this situation can be found using the equation:

$$\mu_o = \frac{m}{n} = \frac{1.5 \cdot 10^{-4} \frac{N \cdot m}{C^2}}{1233 \frac{1}{m}} = 1.2 \cdot 10^{-7} \frac{N \cdot m^2}{C^2}$$

Lastly, using equation 5 we can calculate the percent difference between these experimental permeability constant values and the theoretical value. Doing this calculation results in an 88% difference when using the data from the constant current trial and a 90% difference when using the data from the constant *n* trial. This should be expected since as shown in tables 1 and 2, the theoretical magnetic field values and experimental magnetic field values differed by approximately 90% in every case.

Conclusion:

As stated before, while our data underperformed in respect to the theoretical values in every trial, the trend that it followed matched the trend of the theoretical data within the limits of uncertainty, confirming the equation: $B = \mu_o n I_o$,that when *n* increased the magnetic field of the solenoid increased as well in a linear fashion. Also, as predicted by the equation, our data supported that as I_o increased so did the magnetic field of the solenoid in a linear manner. Still, the data showed very large systematic error in that respect, and this is because we were not able to get our probe to record the magnetic field inside the solenoid, and by holding it outside of the solenoid the field lowered significantly. If we had used larger solenoids, our data would most likely be much closer to the theoretical data in magnitude. Other uncertainty we experienced was most likely caused by our inability to keep the magnetic field probe steady in the exact same place with no movement in the probe whatsoever. Since the strength of magnetic field we were measuring was a relatively small value this altered the reading notably. To cut down on this source of uncertainty, a switch should be introduced into the circuit along with a clamp and post. The clamp and post would keep the probe steadier than our hands would and the switch would be used to open and close the circuit so the solenoid would not have to be moved to and from the probe to take a measurement of the magnetic field reading of the general area.

Works Cited

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