1–D Wave Equation

Let’s look at a wave in a stretched string. We can assume small angles for most situations.

Looking at the motion in the $x$-direction

$$\sum \vec{F}_x = m \ddot{a}_x$$

$$F_R \cos \theta_R \hat{x} - F_L \cos \theta_L \hat{x} = m(0)$$

$$F_R (1) - F_L (1) = 0$$

$$F_R = F_L = F_T$$

This is what we should expect from our experience with strings.
Looking at the motion in the $y$-direction

$$\sum F_y = m \ddot{a}_y$$

$$F_T \sin \theta_R \dot{y} - F_T \sin \theta_L \dot{y} = m \frac{\partial^2 y}{\partial t^2} \dot{y}$$

$$F_T \left( \frac{\partial y(x + \Delta x)}{\partial x} - \frac{\partial y(x)}{\partial x} \right) = \mu \Delta x \frac{\partial^2 y}{\partial t^2}$$

where $\mu$ is the linear mass density.
\[ \frac{\partial y(x + \Delta x)}{\partial x} - \frac{\partial y(x)}{\partial x} \]

\[ \frac{\Delta x}{\Delta x} = \frac{\mu}{F_T} \frac{\partial^2 y}{\partial t^2} \]

In the limit as \( \Delta x \to 0 \)

\[ \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F_T} \frac{\partial^2 y}{\partial t^2} \]

\[ v = \sqrt{\frac{F_T}{\mu}} \]