Physics 425  
Quantum Physics  
Spring, 2016

Meets:  
10:00 am – 11:50 am  
Tuesday, Thursday  
150 Meldrum Hall

Instructor:  
Dr. Christopher Cline  
278 Meldrum Hall  
832-2346  
ccline@westminstercollege.edu

Textbook:  
Required:  
*Introduction to Quantum Mechanics*, 2nd Ed., David J. Griffiths.

Course Description: Physics 425 is a junior/senior level course in quantum mechanics – the study of the behavior of the smallest components of our universe, such as fundamental “particles”, atoms, and molecules. Quantum mechanics is a collection of postulates and consequences thereof, formulated in the language of mathematics, which provides tools for the analysis, prediction, and understanding of observed phenomena in the microscopic domain. As such, this is a course in purely theoretical physics, and so will involve a heavy dose of mathematics, including multi-variable calculus, differential equations, and linear algebra. I will show you how to use these mathematical techniques to solve the physics problems, rather than assuming that you remember them from somewhere else. This course will begin to look a lot like an advanced mathematical methods class; however, remember that the mathematics is only a tool; don’t let it get in the way of the physics.

Many students find quantum physics to be difficult for a number of reasons, not least because it appears to counter so much of the rigor of classical physics that students have worked hard to learn in their early courses in high school and college. Its "essence" appears completely at odds with one's experience and sense of how things "ought" to work. The apparent "counter-intuitiveness" of quantum mechanics has the consequence that it is virtually impossible to develop the subject with the smoothness with which one can Newtonian mechanics, and so many students have said that they just don't "get" quantum mechanics. To a large extent the only real option is to plunge in to the guts of quantum mechanics and trust that the customs of quantum culture will become familiar as one's experience with it grows. Our key goal in this class is to give you the mathematical tools, and the practice, to do quantum mechanics. The logic here is that understanding begins with familiarity. Much private study, calculation, and reflection are necessary to develop a proper sense of how quantum mechanics "hangs together" - there is no one formula or group of magic words to help you "get it" quickly. Mull things over in your own mind, discuss them with your classmates and me, and when you come to understand something, write it down.

Some of you have had an introduction to quantum mechanics in your first course of physical chemistry. Although I think review is a good idea, I do not want to spend all our time treading old ground. So, following a spiral metaphor – where we revisit old material, but with more depth – we will plumb the depths of some of the material you have already covered, but we will push on into new territory too.

My goal is to have minimal lecturing and lots of discussion from your readings. We will also explore the ideas of quantum physics through ConcepTests and Tutorials. Also, I hope to have lots of problem solving. I’ll do some in class; you’ll do lots in class; you’ll do lots more at home.

As you grapple with the concepts of quantum physics, and attempt to make sense of everything, it might help to revisit Richard Feynman’s observation:

“Now we know how the electrons and light behave. But what can I call it? If I say they behave like particles I give the wrong impression; also if I say they behave like waves. They behave in their own inimitable way, which technically could be called a quantum mechanical way. They behave in a way that is like nothing that you have ever seen before. Your experience with things that you have seen before is incomplete. The behavior of things on a very tiny scale is simply different. An atom does not behave like a weight hanging on a spring and oscillating. Nor does it behave like a miniature representation of the solar system with little planets going around in orbits. Nor does it appear to be somewhat like a cloud or fog of some sort surrounding the nucleus. It behaves like nothing you have ever seen before.”  (*The Character of Physical Law*, MIT Press, 1965)
**Learning Goals:** Most of this class focuses on problems in one dimension, although the class also covers problems such as the hydrogen atom, angular momentum, and spin. This list represents what we want you to be able to do at the end of the course:

**Course Scale Learning Goals**

- **Math/physics connection:** Students should be able to translate a physical description of a junior-level quantum mechanics problem into the mathematical equation necessary to solve it. Students should be able to explain the physical meaning of the formal and/or mathematical formulation of and/or solution to a junior-level quantum mechanics problem. Students should be able to achieve physical insight through the mathematics of a problem.

- **Visualization:** Students should be able to sketch the physical parameters of a problem (e.g., wave function, potential, probability distribution), as appropriate for a particular problem. When presented with a graph of a wave function or probability density, students should be able to derive appropriate physical parameters of a system.

- **Knowledge Organization:** Students should be able to articulate the big ideas from each content area, and/or lecture, thus indicating that they have organized their content knowledge. They should be able to filter this knowledge to access the information that they need to apply to a particular physical problem. This organizational process should build on knowledge gained in earlier physics classes.

- **Communication:** Students should be able to justify and explain their thinking and/or approach to a problem or physical situation, in either written or oral form.

- **Problem-solving techniques:** When faced with a quantum mechanics problem, students should be able to choose and apply appropriate problem solving techniques. They should be able to transfer the techniques learned in class and through homework to novel contexts (i.e., to solve problems which do not map directly to those in the book). They should be able to justify their selected approach (see "Communication" above). In addition to building on techniques learned in previous courses (e.g., recognizing boundary conditions, setting up and solving differential equations, separation of variables, power-series solutions, operator methods), students are expected to develop specific new techniques as listed in concept-scale learning goals below.
  - **Approximations:** Students should be able to recognize when approximations are useful, and to use them effectively (e.g., when the energy is very high, or barrier width very wide). Students should be able to indicate how many terms of a series solution must be retained to obtain a solution of a given order.
  - **Symmetries:** Students should be able to recognize symmetries and be able to take advantage of them in order to choose the appropriate method for solving a problem (e.g., when parity allows you to eliminate certain solutions).

- **Problem-solving strategies:** Students should be able to draw upon their knowledge and skills to attack a problem even when a process leading to a correct solution is not yet clear. Students should continue to develop their ability to monitor their progress towards a solution by learning how to:
  - Backtrack and try a new approach when necessary
  - Recognize when a solution has been reached and be able to articulate why this solution is valid (see “Expecting and Checking Solution” below)
  - Persist through to the solution of problems requiring many steps

- **Expecting and checking solution:** When appropriate for a given problem, students should be able to articulate their expectations for the solution to a problem, such as:
  - The general shape of the wave function
  - Dependence on coordinate choice
  - Behavior at large distances
  - Problem symmetry

For all problems, students should be able to justify the reasonableness of a solution they have reached, by using methods such as:

- Checking solution symmetry
- Verifying boundary conditions
- Order of magnitude estimates
- Dimensional analysis
- Limiting or special cases (e.g., checking the solution for correct behavior in limiting or known cases)
• **Intellectual maturity:** Students should accept full responsibility for their own learning. They should be aware of what they do and do not understand about physical phenomena and classes of problem. They should learn to ask sophisticated, specific questions. Students should learn to identify and articulate where in a problem they experienced difficulty and to take appropriate action to move beyond that difficulty. Finally, they should regularly check their understanding against these learning goals and seek out appropriate help to fill in any gaps.

• **Coherent Theory:** Students should recognize that the material covered in this course sets a framework for a consistent and complete understanding of quantum mechanics.

• **Build on Earlier Material:** While the material in the course represents a significant departure from earlier course work both mathematically and conceptually, students should recognize and make use of connections to prior work, techniques and understanding gained in classes in classical physics as well as in their modern physics class.

**Subject Scale Learning Goals**

The goals below pertain to specific areas in the study of quantum mechanics that are to be learned in this course. They are organized by subject and thus do not follow any textbook. The subject categories are:

- Mathematics
- Measurement and the quantum state
- The Schrödinger Equation
- Formalism
- Important Systems
- Scattering
- Angular Momentum and Spin

**Mathematics**

*Prerequisites*

- **Differential Equations:**
  - solve straightforward first and second order differential equations using a variety of methods.
  - recognize when separation of variables will simplify a differential equation and correctly apply the technique.

- **Complex Numbers:** Students should be thoroughly familiar with complex numbers and be able to find the real part, the imaginary part, the complex conjugate and the norm of any complex expression.

- **Linear Algebra:** Given a matrix operator, students should be able to find the eigenvalues and eigenvectors of the operator. Not only be able to diagonalize the matrix but be able to explain the physical significance of the procedure and the result.

- **Hamiltonian Formalism:** Students should be able to set up the Hamiltonian for a classical system.

*Goals*

- **Statistics:** Due to the statistical nature of quantum mechanics, students should be adept at computing probabilities and standard deviations.

- **Dirac Delta Function:** Students should be able to correctly compute integrals that contain one or more Dirac delta functions.

- **Vector Spaces:**
  - Given a set of real or abstract (e.g., Hilbert space) vectors, students should be able to determine whether the set constitutes a vector space.
  - Given a set of real or abstract (e.g., Hilbert space) vectors, students should be able to determine whether or not they form a basis of a given vector space.

- **Hilbert Space:** Students should be able to compute the correct coefficients of a Hilbert space vector given a basis.

- **Operator Theory:** Students should be able to compute the expectation value of an operator in a given state. More generally, compute all the matrix elements of an operator in a given basis. Identify a Hermitian operator.

**Measurement and the Quantum State**

*Goals*

- **The State Vector:**
  - Students should be able to correctly normalize a (normalizable) quantum state.
  - Students should be able to describe and calculate different representations of a quantum state (e.g., position space, momentum space).
• **Observable Operators:**
  o Students will know that observable quantities are represented by Hermitian operators.
  o Given a wave function and an observable operator, students should be able to calculate that operator's expectation value.
  o For simple systems (e.g., 1-D infinite square well), students should be able to find the eigenvectors and eigenvalues for the energy operator.

• **Measurement Predictions:**
  o Given the eigenstates of an operator, students should be able to compute the possible results of a measurement of the observable which corresponds to that operator.
  o Given a quantum state and the eigenbasis of an observable operator, students should be able to compute the probabilities of obtaining the possible values which would result from a measurement of the corresponding observable quantity.
  o Given the results of a repeated measurement of an observable on a quantum state, students should be able to construct a plausible quantum state as a superposition of the eigenstates of the operator associated with the observable.

• **Measurement Effects:**
  o Students should be able to describe what is known about the state of a system immediately after a measurement, including the significance of the measured value.

• **Time Evolution:**
  o Given an initial wave function and a basis of energy eigenstates, students should be able to find the time-dependent wave function.
  o Given an initial wave function and a basis of energy eigenstates, students should be able to deduce when the probability distribution of an operator will be time dependent.

• **Operator Commutation and Compatibility:**
  o Students should be able to describe the relationship that must exist between two operators in order for a common eigenbasis to exist.
  o Students should be able to compute the commutator of the position and momentum operators as well as the commutation relationships between angular momentum operators.
  o Students should be able to describe the effect of following the measurement of an observable with the measurement of an incompatible operator.
  o Given two non-commuting observables, A and B and the result of a measurement of A, students should be able to compute the possible outcomes of a subsequent measurement of B along with the appropriate probabilities.

**Schrödinger Equation**

Goals

• **Time Dependent Schrödinger Equation:** Student should be able to use the time dependent Schrödinger Equation to compute the time evolution of a wave function.

• **Time Independent Schrödinger Equation:** Students should be able to describe the conditions under which separation of variables can be used to create a time independent Schrödinger Equation and use this equation to:
  o Students should be able to solve for the energy levels of the system
  o Students should be able to apply boundary conditions and solve for the stationary states (energy eigenstates) of the system
  o Students should be able to apply the Hamiltonian and boundary conditions to determine whether the energy eigenstates are discrete or continuous.
  o Students should be able to specify the evolution in time of a system when both an initial state and the energy eigenstates known

**Formalism**

Goals

• **Normalization:**
  o Students should be able to explain the relationship between the normalization of a wave function and the ability to correctly calculate expectation values or probability densities.
  o Students should be able to correctly normalize any wave function that represents a physically realizable state.

• **Hamiltonian:**
  o Students should be able to set up the Hamiltonian for a quantum mechanical system when they can calculate the potential energy for the corresponding classical system.
Students should be able to use commutation relations to be able to determine which operators have eigenstates that are time independent.

- **Uncertainty Principle:**
  - Given a quantum state and an observable, students should be able to compute the uncertainty (standard deviation in the measurement) of the observable.
  - Given two observables, students should be able to compute the minimum uncertainty of measuring both observables on any quantum state.

- **Probability in Quantum Mechanics:**
  - Given a (time-dependent) wave function, students should be able to compute the time-dependent probability density.
  - For a given quantum state, students should be able to compute the probability of measuring any particular value for any common observable.

### Important Systems

#### Goals

- **Infinite Square Well:** Students should be thoroughly familiar with all aspects of the one dimensional infinite square well.
  - Given the size and position of the potential, students should be able to compute the energy eigenvalues and the energy eigenstate position-space wave functions.
  - Students should be able to compute the time evolution of a superposition of energy eigenstates as well as the expectation value of common observables for a superposition state.

- **General One-dimensional Systems:**
  - Given a one-dimensional potential, students should be able to sketch the first few energy eigenstates.

- **Finite Square Well:** Students should be able to sketch the wave function for a system with one or more finite square wells. They should be able to qualitatively predict the time evolution of the wave function given an initial state.

- **Harmonic Oscillator:**
  - Given a specific harmonic-oscillator potential, students should be able to compute the energy eigenvalues.
  - Given the raising and lowering operators, students should be able to find the lowest energy eigenstate.
  - Given the raising and lowering operators and an energy-eigenstate wave function, students should be able to find the energy eigenstates on either side.
  - Students should be able to sketch the first few energy eigenstates of the harmonic oscillator.
  - Students should be able to compute position and momentum expectation values using the raising and lowering operators.

- **Free Particle:** Students should be adept at using the position-space and momentum-space wave functions of the free particle. In particular, use them to construct wave packets.

- **Hydrogen Atom:**
  - Students should be able to set up the Schrödinger equation for a hydrogen-like atoms.
  - Students should be able to perform variable separation on the Schrödinger equation for a hydrogen-like atoms.
  - Students should be able to describe the energy eigenstates for hydrogen-like atoms including the significance and use of their quantum numbers.

- **Two-State Systems:**
  - Given a two-dimensional Hamiltonian, students should be able to find its eigenstates and eigenvalues.
  - Given a two-state system in a superposition state, students should be able to correctly compute the probabilities of measuring each eigenvalue.

### Angular Momentum and Spin

#### Goals

- **Angular Momentum in Quantum Mechanics:**
  - Students should be able to compute the angular momentum of a system in a known eigenstate of an angular momentum operator (e.g., $L^2$, $L_z$).
  - Given a system in a known state, students should be able to compute the probabilities of the possible results of measuring an angular momentum observable (e.g., $L^2$, $L_z$, $L_x$).

- **Spin:**
  - Given a system in a known state, students should be able to compute the probabilities of the possible results of measuring a spin observable (e.g., $S^2$, $S_z$, $S_x$).
Conditions of enrollment: Physics 212 (Physics for Scientists & Engineers II), Physics 309 (Mathematical Methods of Physics), Math 203 (Multivariate Calculus), and Math 211 (Linear Algebra) are prerequisites for all students enrolled in this course.

How to get help: My office hours are MTWTh 1:00 pm-3:00 pm. If you can’t come during any of these hours, I will be happy to make an appointment with you for another time. For me, the most enjoyable aspect of teaching is working with students one-on-one. Please, please come see me often—especially if you run into difficulties with concepts.

Class Attendance and Participation: Class meetings are TTh 10:00am-11:50am. Preparation for class, attendance, and participation will be rewarded.

Course Requirements

Reading Memos: It is nearly useless to read a physics text as you would a novel. “Studying” such a text requires that you be an active reader, that you remain engaged in a virtual and appropriately skeptical conversation with the author. You should, for example: (1) reserve doubt about everything the text says until it thoroughly convinces you, (2) think about situations to which the author’s arguments might not apply, (3) make notes in the margins, (4) draw your own sketches and graphs to help visualize situations and functional behaviors, and especially (5) fill in all of the missing steps in any mathematical arguments. Indeed it is all too tempting to simply take the author’s word for everything including the results of any calculation; after all, he or she wouldn’t consciously lie to you, right? Well, yes; probably. But if you get into that habit, you will become a passive reader. Your mind forms no permanent “hooks” on which to store the information being presented. The time spent in the process may well be reduced, but will also have been essentially wasted.

Perhaps mathematician Paul R. Halmos gave the best advice about how to study: “Study actively. Don’t just read the text; fight it! Ask your own questions, look for your own examples, discover your own proofs.” (I Want to Be a Mathematician, New York: Springer-Verlag, 1985)

Accordingly, in order to help you form or hone these important good study habits, I will ask you to produce and turn in a “Reading Memo” at the beginning of each day for which there is assigned reading. A “Reading Memo” is an informal running collection of thoughts about and reactions to the material in the text. As you study, keep a pencil in hand and note any questions that occur to you; any surprises, insights, or connections to other things you know about; anything you think may be wrong or incomplete and why you think so; anything you think could be said more clearly and your proposed revision; etc. Beyond their effectiveness at helping you to stay engaged as you study, your Reading Memos will also help me to understand those items and topics that may require more attention in class.

Your Reading Memos will be given full (2 pts) or half credit (1 pt) purely on the basis of whether or not it appears that your good faith effort was involved and not at all on the basis of format, sophistication, vocabulary, correctness, etc. In order to allow for extraordinary circumstances (including absence for any reason), I will throw out up to three “missing” Reading Memos.

Homework: I will make regular Homework Assignments due at intervals of very approximately a week and a half to two weeks at the beginning of a specified class meeting.

As you surely know by now, the primary purpose of assigned problems in physics is absolutely not to see if you can get the right answer. Rather, it is for you to practice and then demonstrate that you have learned 1) how to determine the fundamental physical principles that are involved in a described situation and 2) how to apply those principles in a disciplined and orderly fashion. Of course, if you have learned how to do these things, you should expect to get the right answer too, but that is - really - of secondary importance. You will find - indeed, you probably have found - that, given time, an open book, lots of worked examples, and knowledge of the correct answer, it is very often possible to "get the answer" without the slightest understanding of what you are doing. Please guard against this; it is a complete waste of your time because it does not prepare you for, and it obviously will not work on, exams.

Accordingly, we are not - and you should not be - satisfied with problem "solutions" that simply consist of a series of mathematical manipulations leading to a result. Instead, the problem solutions you submit are to be "presented." By this we mean that they should be readable by someone who does not have access to the problem statement; should include written explanations and thoughtful comments about what you are doing and, especially, why; should use well-defined and consistent notation (employing unique and meaningful subscripts and superscripts as necessary); should be accompanied by neatly drawn and carefully labeled diagrams; and should flow in a logical and orderly progression down the page. They should use more space for the written explanatory information than for the mathematics! They should not include lengthy, multiple-step, purely mathematical manipulations because it only serves to obscure the physics. Do this
kind of work on scratch paper and simply say something like "Solving equations 1, 2, and 3 for x, y, and z, we obtain ..." and give the result.

I will not “check” your homework solutions in any serious fashion; it is up to you to check them against the solutions that I will hand out and to get answers—from me or others in the class—to any remaining questions you have. I will look over your work only casually and assign a holistic score of 1 to 4 with 4 meaning that the problem set appears to be exceptional—complete, very well presented, and mostly correct; 3, good—at least nearly complete, clearly presented, and pretty much correct; 2—incomplete or not so clearly presented; 1—not a good faith effort. Unsubmitted problem sets will receive a 0.

I strongly encourage you to form study groups and to discuss with others your readings, questions that come up in and out of class, and how to go about solving problems. The work you turn in, however, must be yours, based on the understanding you have acquired. When faced with two write-ups that show any signs of copying, I conclude that at least one person hasn’t done the work. In such cases both papers will receive no credit.

I do not accept late Homework Assignments, but, in order to allow for extraordinary circumstances (including absence for any reason), I will throw out your two lowest scores.

Subjective Bonus: A small portion of your grade is also determined by my own overall subjective evaluation of your work in the class. Although it is subjective, my policy is that it will not be less than the average of your Reading Memo and Homework scores. It allows me only to reward students who make contributions to the class that may not be fully recognized, who make particularly effective use of office hours, or who, in any other way, seem to deserve a bit of additional credit.

Midterm and Final: There will be two take-home exams, including the final. You may use your textbook and notes that you generated during the course. You may not work with or gain assistance from anyone except members of the Westminster physics faculty. Of course, I trust you will do all your own work on the exams. If you are caught cheating on an exam you will receive an F for the exam for the first offense; for a second offense, an F for the entire course.

Grading: Your overall “Course Score” will be calculated using the following relative weights:

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<tr>
<th>Component</th>
<th>Weight</th>
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<tr>
<td>Reading Memos</td>
<td>10%</td>
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<td>Homework</td>
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<td>Subjective Bonus</td>
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<tr>
<td>Exams</td>
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Academic Integrity: Please make sure that you have read and fully understood Westminster's Policy on Academic Honesty (and Dishonesty) that appears in the Academic Catalog. My sincere desire is to act as facilitator—not an enforcer—for your studies in physics. Accordingly, I operate on the assumption that all of our interactions are based on openness, honesty, and good faith. I expect all of us to be honest and to treat each other fairly and with respect. Because our trust in each other is absolutely crucial to the effectiveness of our relationship, I take an uncompromising stance, as should you, on the necessity for sanctions when it is violated.

Services for Students with Disabilities: Westminster College is committed to providing a working and learning atmosphere that reasonably accommodates qualified persons with disabilities. If you have any disability that may affect your access to this course, please contact the Office of Disability Services (DS), specifically Ginny DeWitt, Disability Services Coordinator, in the START Center (801-832-2280). Reasonable academic accommodations are reviewed for all students who have qualified, documented disabilities. Services are coordinated with the student and instructor by the DS Coordinator. Westminster College provides reasonable access to courses but this does not necessarily equate to ensuring your success in any course. If you need assistance or if you feel you have been unlawfully discriminated against on the basis of disability, you may seek resolution through established grievance policy and procedures by contacting the ADA Coordinator (Jason Sweat, 801-832-2657, jsweat@westminstercollege.edu, Gore 213) and/or the Office of the General Counsel at 801-832-2565.

Additional information about disabilities service guidelines is on the Disabilities Services web page. If you have questions regarding services for students with disabilities or require alternate format of this information, please call 832-2280, TTY 832-2286, or email startcenter@westminstercollege.edu.

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based discrimination and harassment, sexual harassment, sexual misconduct, sexual assault, rape, stalking, dating violence, domestic violence, sexual exploitation, and any other form of sexual or interpersonal violence. The Policy extends not only to students of the college but also to employees. The Policy is available at the Title IX web page and discusses your rights, the process for investigating complaints, and sanctions for violations of the Policy. The Policy strictly prohibits retaliation against anyone who reports or participates in an investigation regarding alleged or suspected violations of the Policy. Westminster’s Title IX Coordinator is Jason Schwartz-Johnson. Jason can be reached at 801-832-2262, jsj@westminstercollege.edu, or in Malouf 107. The Policy has additional support services and resources as well. Please note that to the extent permitted by law, the College aims to protect the privacy of all parties involved in the investigation and resolution of alleged or suspected violations of the Policy. However, the College has a duty to investigate and take remedial measures in response to complaints and cannot guarantee confidentiality. As an instructor I am also required by our school to report incidents of gender-based discrimination or harassment, sexual harassment, sexual misconduct, or other forms of sexual or interpersonal violence to the Title IX Coordinator and thus cannot guarantee confidentiality.

**Title VI:** Title VI of the Civil Rights Act of 1964 prohibits discrimination based on race, color or national origin in any program or activity receiving federal financial assistance. The Department of Education has interpreted Title VI as prohibiting racial harassment, and such harassment is prohibited in all facets of campus life at Westminster College. If you encounter this type of discrimination/harassment, you can contact the Office of the General Counsel at 801-832-2565.