UNIT 5: ONE-DIMENSIONAL FORCES, MASS, AND MOTION

The Apollo/Saturn V space vehicle carrying Apollo 11 astronauts Neil A. Armstrong, Michael Collins, and Edwin E. Aldrin, Jr., lifted off at 9:32 a.m. EDT on July 16, 1969. This was our nation’s first manned lunar landing mission. The 36-foot-high vehicle generated a thrust of seven and one-half million pounds during liftoff. It lumbered off the launch pad very slowly at first, and then picked up speed rapidly. Its velocity may have increased at slightly more than a constant rate during the early stages of take off. This increasing acceleration was probably followed by a decreasing acceleration even if the ejected fuel created a constant thrust force on the rocket. Yet, the rocket’s motion and escape from the Earth’s gravitational attraction happened in accordance with Newton’s Laws of motion. How is it possible for the rocket to give itself a constant thrust force and have an acceleration that is not constant? As you study the fundamental relationships between one-dimensional net force, mass, and motion in this unit, you should be able to answer this question.
UNIT 5: ONE-DIMENSIONAL FORCES, MASS, AND MOTION

If you find the study of motion difficult, reflect that it took mankind . . . over sixteen hundred years to reach a clear understanding of motion; you should hardly be impatient if it takes you several weeks.

Eric Rogers (1961)

No one must think that Newton’s great creation [the three laws of motion] can be overthrown . . . His clear and wide ideas will forever retain their significance as the foundation on which our modern conceptions of physics have been built.

Albert Einstein (1948)

OBJECTIVES

1. To devise a method for applying a constant force to an object.

2. To find a mathematical relationship between force and motion.

3. To devise a force scale to measure one, two, three, etc. units of force.

4. To understand how different one-dimensional forces acting along the same line can be combined.

5. To develop a definition of mass in terms of an object’s motion under the influence of a known force.

6. To combine all of the observations and develop statements of Newton’s First and Second Laws of Motion for one-dimensional motion with very little friction present.
5.1 OVERVIEW

So far in your study of one–dimensional motion you have learned to observe and describe motion in several ways. The next step is to study the causes of motion.

The motion of an object is obviously influenced by pushes or pulls, electrical or magnetic attractions, winds, and so on. Even casual observations tell us that the way an object that is pushed or pulled moves depends on the “amount of stuff” it is made of. It’s easier to push a shopping cart than a Mack truck. In physics we usually refer to a push or pull as a force, while we refer to the “amount of stuff” as the mass of an object. In this unit you will explore intuitive ideas of force and mass, and study the influence of force on motion. Finally, you will formulate two laws of motion developed by Isaac Newton in the seventeenth century.

Newton’s Laws of motion are powerful! When forces on a system are known, Newton’s Laws can be used to describe or predict its behavior. This predictive ability is of tremendous importance to engineers who want to design bridges that don’t collapse and cars that stop reliably. Also, an understanding of the Laws of Motion allows scientists to deduce the nature of fundamental forces such as intergalactic forces and nuclear forces on the basis of observations of motions. As Newton stated,

… the phenomena of motions [can be used] to investigate the forces of Nature, and then … these forces [can be used] to demonstrate other phenomena...the motions of the planets, the comets, the moon and the sea.

Note: The classical laws of motion that we will develop in this unit provide for all practical purposes “exact” descriptions of the motions of everyday objects traveling at ordinary speeds. During the early part of the twentieth century two new theories were developed—quantum theory, which describes motion in the atomic realm, and relativity, which describes objects moving extremely fast. Once you master the classical description of ordinary motions, it is exciting indeed to see how these laws are modified so that they will also describe very small objects or objects moving at extraordinary speeds (close to that of light).

*Often when forces are known, the actual position and velocity of the system can be predicted within the limits of experimental uncertainty. However, there are some systems that exhibit chaotic behavior that can be described in terms of well understood forces, but not predicted. In Unit 15 you will study a chaotic system.
FORCE AND MOTION

5.2 MOTION FROM A CONSTANT FORCE

In your previous study of motion in this course you concentrated on describing motion rather than on understanding its causes. From your experiences, you know that force and motion are related in some way. For example, to start a bicycle moving, you have to push down on the pedals, or to start a wagon moving you have to pull on it. What kinds of forces lead to steady motion? To changes in motion?

Before you can study the relationship between force and motion, you must be able to devise a useful definition of force and develop reliable ways to measure it. Then you can investigate these relationships by applying forces of different strengths to various objects. You can begin by figuring out how to apply a constant force to a person who can slide along a smooth floor or roll along on a low friction cart. Then you can measure the motion of the person under the influence of a constant force using a computer-based laboratory system.

In order to do the activities in this section you will need some but not necessarily all of the following items to investigate the creation of a constant force:

- 1 rod
- 1 table clamp
- 1 ruler
- 1 meter stick
- 1 large rubber band, #117 (3.5” × 0.75”)
- 1 large spring scale, about 15 kg
- 1 mass pan, 1 kg
- 2 masses, 1 kg
- 3 masses, 2 kg

These items can be used to relate force and motion:

- 1 computer data acquisition system with a motion sensor
- 1 large plastic garbage bag
- 1 Kinesthetics cart (or large skateboard)

Recommended Group Size: 2
Interactive Demo OK?: Y

What Is Force?

What is force and how is it measured? The word “force” is a very common part of everyday language. One of the major tasks in this unit is to help you move in stages from an informal understanding of the meaning of the term force as a push or pull to a more precise, quantitative definition that is useful in relating force to motion.

*Recall that a computer data acquisition system consists of a sensor, an interface, a personal computer, and data collection software.
5.2.1. **Activity: Ideas about Force**

Attempt to define the word force in your own words. What are some examples of forces? How might you measure how large a given force is?

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**Creating a Constant Force**

In order to explore the relationship between force and motion, you should try to figure out how to apply a constant force to an object. You can use any of the equipment listed for this section or any other common items you have available.

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5.2.2. **Activity: Applying a Constant Force**

Devise a method for pulling on an object with a constant force. Explain your method and also explain why you believe that the force is constant.

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**Predicting Motion from a Constant Pull**

Suppose you exert a fairly large constant pull on a person? We are interested in having you track the motion of a person sitting directly on a smooth level floor (or on top of a large garbage bag). Then we would like you to track the motion of a person riding on a low friction Kinesthetic Cart or skateboard. A computer based data acquisition system can be set up to track the motion with a motion sensor placed behind the person being pulled. These situations are shown in Figure 5.2.
5.2.3. Activity: Predicting the Velocity of a Person Being Pulled with a Constant Force

a. Consider the situation in Figure 5.2a. What do you predict you might see for a velocity vs. time graph if the person starts from rest and slides along the floor while being pulled away from a motion sensor with a constant applied force? Sketch the shape of the predicted graph in the space below.

b. Consider the situation in Figure 5.2b. What do you predict you might see for a velocity vs. time graph if the person starts from rest and rolls along the floor while being pulled away from a motion sensor with a constant force? Sketch the shape of the predicted graph in the space below.
c. Explain the reasons for your predictions. If you predict that the two graphs will have the same shape, explain why. If you predict different shapes, explain why you expect the shapes to be different.

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**Observing Motion from a Constant Pull**

You can work with the rest of the class or with one or more partners to create and record the sliding and rolling motions with a constant force using a computer-based laboratory system and motion detector. We suggest you use a large rubber band or spring scale stretched out to a constant distance to create a constant pulling force. If the person being pulled holds a meter stick, the puller can try to keep the stretch fairly constant. The person should be pulled away from the motion sensor. These motions take some practice to create. To get reliable measurements, apply enough constant force in each case to get velocities of 0.5 m/s or more for a time period of about 5 seconds.

**5.2.4. Activity: Observing the Velocity of a Person Being Pulled with a Constant Force**

a. Create the sliding situation depicted in Figure 5.2a and observe what the velocity is as a function of time. Fill in the horizontal and vertical axis values on the following graph frame and sketch the observed graph.
b. Examine the overall shape of the graph for the sliding motion. (Ignore the smaller bumps associated with the wobbling spring or rubber band.) Is the velocity zero, constant, or changing? Is the acceleration zero, constant, or changing? Explain what characteristic of the graph shape supports your description.

c. Explain how you created the constant force. Describe the pulling method used. If you stretched the rubber band or spring, what was the amount of the stretch?

d. Create the one-dimensional rolling situation depicted in Figure 5.2b and observe what the velocity along the $x$ axis is as a function of time. Fill in the horizontal and vertical axis values on the following graph frame and sketch the observed graph.

![Observed rolling motion graph](image)
e. Examine the overall shape of the graph for the rolling motion. (Ignore the smaller bumps associated with the wobbling spring or rubber band.) Is the velocity zero, constant, or changing? Is the acceleration zero, constant, or changing? Explain what characteristic of the graph shape supports your description.

f. How did the observations you reported in a. and d. compare with your predictions?

g. State a general rule based on your observations for the relationship between a constant applied force and velocity when friction is significant, for example, when an object slides.

h. State a general rule based on your observations for the relationship between a constant applied force and acceleration when the friction is low.
Based on your observed velocity vs. time graph for the low friction rolling motion resulting from a steady pull, sketch an acceleration vs. time graph for the time period during which the pulling force was roughly constant.

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**Rolling vs. Sliding Motions**

You probably observed that a constant force leads to a constant velocity when an object is sliding. On the other hand, a rolling object appears to move at a constant acceleration under the influence of a constant force. This is a surprising observation to most people! The sliding motion involves a significant amount of friction while the rolling does not. In this unit, you will be working with rolling objects that have only a small amount of friction to discover Laws of Motion in the absence of friction. In Unit 7 you will return to the question of how friction can be incorporated into the laws of motion.

**5.3 COMBINING EQUIVALENT FORCES**

In the last section you should have discovered that a single constant force applied to an object that rolls without much friction causes it to move with a constant acceleration. What happens to the acceleration of a rolling object when the force doubles or triples? What if the force is not constant? What happens to the object’s acceleration then?

Before you can proceed with further investigations of the relationship between force and motion, you need to explore reliable ways to measure arbitrary forces. *Being able to produce and combine equivalent forces will enable you to define a force scale and understand how forces of arbitrary strengths can be measured and created.*

We’d like you to work in small groups to investigate how to find equivalent force and combine them. You will use small low-friction dynamics carts. Thus, you need to learn to work with a smaller force than the one used to pull a student riding on a Kinesthetics cart.
The activities in this section are not completely specified. Although the following items should be available, you might not need all of them:

- 2 table clamps
- 2 rods
- 1 ruler
- 6 rubber bands, #14
- 2 mass pans, 50 g
- 2 masses, 50 g
- 5 masses, 100 g
- 1 spring scale, 10 N

Equivalent Forces

A way to create double and triple forces is to combine equivalent forces. You should start this investigation by pulling a single rubber band out to some predetermined length that you choose. You can name the unit of force associated with the pull after yourself or make up another name for it.

5.3.1. Activity: Creating Equivalent Forces

a. Pick out one of the #14 rubber bands as your standard rubber band. You may want to identify it by marking it with a pen or pencil. Now use your rubber band to define your own unit of constant force. Explain how your unit of force is created. In other words, if you were to give your rubber band to someone else, explain how they could use it to pull on something with your unit of constant force.

b. Consult with your partner(s) and decide what you want to call your unit.
c. Suppose we define an equivalent force as a force that accelerates a cart in exactly the same way that your unit of force does. Consult with your partner(s) and think of as many different techniques as possible to create an equivalent force using different objects than your special rubber band. Describe three or more of these techniques.

d. Now consult with your partners and think of two or more ways to test whether a proposed equivalent force is actually equivalent to your force unit.

e. Now create what you think is an equivalent force. Does the “equivalent” force pass the tests you have devised in part d. for equivalency? Explain.

Evaluating Alternatives for Testing Equivalent Forces

Sometimes when there are alternative ways to accomplish a goal some seem better than others. You should discuss your ideas and findings with your classmates and then choose what you think is the best technique for creating a force that is equivalent to your unit in terms of its ability to accelerate a cart in the same way that your force did.
5.3.2. Activity: Equivalent Force Techniques

a. Are there any other tests for equivalency suggested by classmates that you and your partners didn’t think of? If so, list them below.

b. Which do you think is the most scientifically sound test for equivalency? Which of the techniques do you prefer to use to create an equivalent force? Why do you prefer it? How do you know it is valid?

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Combining Equivalent Forces

Since the goal in this section is to learn produce and measure forces of arbitrary strengths, you can start by combining equivalent forces to create a force scale. Let’s assign an x axis that is parallel to the line of motion you are studying. You can then denote your basic unit of force as \( F \).

5.3.3. Activity: Creating \( F, 2F, 3F, \ldots \)

Describe how you could combine your equivalent forces to create forces that have double and triple the strength of your initial force unit.

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5.4 USING STANDARD UNITS TO MEASURE FORCE

So far you have been measuring forces in your own units using procedures that you and your partners have defined. If you measure forces and want to have scientists in another location understand your results, it would be convenient if everyone used the same force unit. The accepted standard unit for force is the newton. The newton is defined as the force that is needed to
give a 1 kilogram mass an acceleration in which the velocity of an object increases by 1 meter per second each second. We’re getting ahead of ourselves because we haven’t defined the kilogram as a unit of mass yet. We will soon.

A scientifically rigorous way to measure a force of 1.0 newtons is to take a 1.0-kg object that can move without friction and apply just the right force to it to get an acceleration of 1.0 m/s/s. That force would, by definition, be one newton—at least to two significant figures. But it is a pain to have to go to all that trouble, and you can use a standard device for measuring force in newtons instead.

The most common device for measuring forces in newtons consists of a spring with a scale attached to it that is marked off in newtons. In theory, someone already figured out how much the spring has to stretch to get a 1.0-kilogram mass moving with an acceleration of 1.0 m/s/s and combined forces to define the appropriate scale.

Another less common but very useful way to measure force is to use an electronic force sensor attached to a computer data acquisition system that has been calibrated to read in newtons.

In the activities in this section, you will measure forces in newtons with both a spring scale and an electronic force sensor using the following equipment:

- 1 ruler
- 6 rubber bands, #14
- 1 spring scale, 10 N
- 1 computer-based laboratory system
- 1 force sensor
- 1 motion software

| Recommended Group Size: | 2 | Interactive Demo OK?: | N |

Measuring Force in Newtons with a Spring Scale

You should devise a way to use rubber bands to show that the forces indicated on a spring scale in newtons are proportional to your rubber band units.

5.4.1. Activity: Converting Your Units to Newtons

a. Devise a way to show that the forces indicated by a spring scale in newtons are proportional to your rubber-band units. Explain what you did and show your data and graph in the space below.
b. Find a conversion factor between your personal rubber-band units and newtons. Explain how you determined the factor and show any calculations in the space below.

Calibration of an Electronic Force Sensor

It is very useful to be able to read forces in newtons using an electronic force sensor. Some types of electronic force sensors require calibration before they can record forces in newtons. In general, calibration involves finding a relationship you or a computer program can use to convert readings on the measuring instrument to the quantity you want to measure. Procedures for force sensor calibration are either built into the motion software or require a mechanical adjustment in the sensor. To calibrate your force sensor using the spring scale as a standard:

1. Set up your computer data acquisition system with a force sensor, motion detector, and interface.
2. Calibrate the force sensor. (You should refer to your sensor manual or software help routine for calibration instructions.)

5.4.2. Activity: Calibrating the Force Sensor in Newtons Using a Spring Scale

a. Use a 5.0 N force as indicated on the spring scale to calibrate your force sensor. What do you think are the major sources of uncertainty in your procedure?

b. Check the accuracy of your calibration, pull on the force sensor with a spring scale that reads 3.0 N of force. Is your calibration accurate? If not, repeat part a.
c. Select a force-time graph. Push and pull on the force sensor and look at the reading on your graph. Is one newton a very large force? Explain.

d. What is the largest pulling (or positive) force you can measure before the force-time graph flattens out, indicating that the force sensor-computer system has been driven beyond its limits?

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**RELATING ACCELERATION AND FORCE**

### 5.5 MEASURING ACCELERATION AS A FUNCTION OF FORCE

You have already determined that, for a situation in which friction is small, a constant force on an object causes it to accelerate at a constant rate. Now that you have a more thorough understanding of how forces can be defined and measured, you are ready to investigate the relationship between the magnitude and direction of the applied force and magnitude and direction of the resulting acceleration for different forces on a moment-by-moment basis.

![Fig. 5.6. How does the measured acceleration along an x axis of a low-friction cart change when the applied force on it is changed?](image)

### 5.5.1. Activity: Predicting Acceleration or Velocity vs. Force

a. Suppose you push and pull on a force sensor attached firmly to a low-friction cart and obtain a graph of force vs. time like that shown on the next page. Do you expect the velocity vs. time or the acceleration vs. time graph to have the same shape as the force vs. time graph does? Explain the reason for your prediction in light of observations you have already made when applying a constant force to a person on a cart.
b. Consider the previous graph that shows sample force vs. time data. Sketch the predicted shape of the graph of corresponding velocity vs. time or acceleration vs. time. Please label the graph’s vertical axis.

In order to investigate how acceleration and force are related, you will push and pull on a force sensor that is attached to a cart and record the motion of the cart. You will need:

- 1 small low-friction dynamics cart
- 2 masses, 500 g (to add mass to the cart)
- 1 smooth ramp or level surface 1–3 meters long
- 1 computer-based data acquisition system
- 1 force sensor (with a hook on its sensitive end)
- 1 adapter bracket (to attach a force sensor to the cart)
- 1 spring scale, 10 N (to calibrate the force sensor)

Recommended Group Size: 2  Interactive Demo OK?: Y
To do the next activity you want to be able to push and pull on a cart while measuring both the force and motion continuously. You can start by attaching a force sensor (with a hook on its end) firmly to a cart. Then you should add about 1.0 kg of mass to the cart and place it on a smooth level track or surface.

![Setup showing a motion sensor tracking the acceleration of a cart rolling on a level track as a force sensor detects the pushes and pulls on it. As usual, the cart must be at least 0.5 m away from the motion detector at all times.](image)

Fig. 5.8. Setup showing a motion sensor tracking the acceleration of a cart rolling on a level track as a force sensor detects the pushes and pulls on it. As usual, the cart must be at least 0.5 m away from the motion detector at all times.

If needed, calibrate the force sensor to read forces between 0 and 5 newtons. Set up the motion software to display two graphs: Force vs. time and acceleration vs. time for about 5 seconds.

### 5.5.2. Activity: Measuring Acceleration vs. Force

1. Zero the force sensor and record data as you grip the hook on the end of the force sensor firmly to push and pull the cart back and forth on the track smoothly. Repeat this process until you have smooth reliable data. Then sketch the force and acceleration graphs. **Note:** If your acceleration graph seems rough and has spikes on it, you should set the averaging for the velocity and acceleration data at something between 5 and 15 points so that small uncertainties in data do not appear on the acceleration graphs.
b. You should have observed that the force and acceleration graphs do indeed have the same shape. Is this what you predicted in Activity 5.5.1?

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**Are Force and Acceleration Proportional to Each Other?**

The fact that the graphs of force and the resulting acceleration caused by it have the same general shape suggests that force and acceleration might be proportional to each other. Recall that if two variables are proportional, a graph of one variable as a function of the other is a straight line passing through the origin, and that the equation relating them would have the form

\[ F_x = ma_x \]  \hspace{1cm} (5.1)

where \( F_x \) represents the force acting, \( a_x \) represents the acceleration, and \( m \) represents the slope of the graph and is the constant of proportionality. Since you have a computer-generated table of values of \( a_x \) and \( F_x \) for a whole series of times, you can use the data you just gathered to test for proportionality between \( a_x \) and \( F_x \).

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**5.5.3. Activity: Finding the Mathematical Relationship Between Acceleration and Force**

a. Use the analysis feature of your motion software or display a data table to obtain about five sets of \( a_x \) and \( F_x \) values. Two significant figures will be fine. **Hint:** Find the corresponding values of \( F_x \) when \( a_x \) is the most negative, approximately zero, and the most positive. Then find a couple of sets of \( a_x \) and \( F_x \) values when \( a \) is between the most negative and zero and between the most positive and zero.

<table>
<thead>
<tr>
<th>( a_x ) (m/s/s)</th>
<th>( F_x ) (N)</th>
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<tbody>
<tr>
<td>1</td>
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b. Create a graph of \( F_x \) vs. \( a_x \) with properly labeled axes including units and affix it on the following page. **Note:** It is conventional to plot the independent variable on the \( x \)-axis and the dependent variable on the \( y \)-axis. Using this convention, you should graph \( a_x \) as a function of \( F_x \) because \( F_x \) is the independent variable that you can change at will as you push or pull on the cart. However, it is more convenient in later activities if you graph \( F_x \) as a function of \( a_x \) instead.
c. Graphs of experimentally determined relationships are seldom perfect since there is usually some scatter due to uncertainties. Taking this fact into account, does the relationship between \( F_x \) and \( a_x \) appear to be a proportional relationship? Why or why not?

d. If the relationship is proportional, what is the constant of proportionality (i.e., the slope of the graph)? Explain how you determined the slope and include units for it. (Fitting, mathematical modeling, drawing a best slope by hand and estimating its value, etc.)

e. Write the general equation that relates \( F_x \) and \( a_x \) in terms of the symbol, \( m \), which represents the slope of the graph.

You have just discovered a one-dimensional law of proportionality between an applied force and acceleration for the situation in which there is almost no
friction. This is almost, but not quite, one of Newton’s famous laws of motion. Before you can enrich the law of proportionality, you will need to explore how the properties of the object being accelerated affect the proportionality constant. In addition, you need to learn about what happens when more than one force acts on an object at the same time.

5.6 NET FORCE: ADDING AND SUBTRACTING FORCES

In this section we will consider what happens when more than one force acts on an object along the same line at the same time. We have not yet thought formally about how forces combine. In doing so, it is useful to treat forces as mathematical entities. Let’s postulate that one-dimensional forces behave mathematically like vector components.

A one-dimensional vector component is a mathematical entity that has both a direction along an axis and a magnitude. If we choose an $x$-axis to lie along the same line as the one-dimensional forces we are considering, then a vector component can have a direction along the positive $x$-axis or a direction along the negative $x$-axis. The magnitude or strength of a one-dimensional force can be represented by a single number, while the direction of a force can be expressed in terms of a plus or minus sign in front of the number representing the strength of the force. One-dimensional vector components can be represented as the product of their magnitude and direction along an $x$-axis as shown below.

**1D forces represented as vector components having the same direction**

\[ F_{Ax} = +5.5 \text{ N and } F_{Bx} = +3.4 \text{ N} \]

or \[ F_{Ax} = -5.5 \text{ N and } F_{Bx} = -3.4 \text{ N} \]

**1D forces represented as vector components acting in opposite directions**

\[ F_{Ax} = +2.3 \text{ N and } F_{Bx} = -7.7 \text{ N} \]

or \[ F_{Ax} = -2.3 \text{ N and } F_{Bx} = +7.7 \text{ N} \]

**Note:** When a vector component is used to represent a physical quantity, we always include the units. In this case, the newton or N is used for force.
You can do some simple observations to determine whether or not one-dimensional forces behave like vectors. To do this you will need:

- 3 identical spring scales, 10 N
- 1 small low-friction cart

| Recommended Group Size: | 2 | Interactive Demo OK?: | Y |

5.6.1. Activity: Do 1D Forces Behave Like Vectors?

a. Describe what happens when a spring scale is hooked to one end of a resting cart and extended in a horizontal direction so that its force is equal to 2.0 N in magnitude. The force points along the positive \(x\)-axis. Does the cart move? If so, how and in what direction? This should be a casual observation—no need to take any data.

![Fig. 5.10.](image)

b. Draw an arrow that represents a scale drawing of the magnitude and direction of the force you are applying. Let one centimeter of arrow length represent each newton of force. Label the arrow with an \(F_{Ax}\) and indicate whether \(F_{Ax} = +2.0\) N or \(-2.0\) N.

c. Observe what kind of motion results when two spring scales are hooked to opposite ends of the cart and extended in a horizontal direction so that each of their forces is equal to 2.0 N in magnitude but opposite in direction. Does the cart move? If so, how and in what direction. What is the combined or net force on the cart? Indicate whether \(F_{Bx} = +2.0\) N or \(-2.0\) N.

![Fig. 5.11.](image)

d. Draw arrows that represent a scale drawing of the magnitudes and directions of the forces you are applying. Let one centimeter of arrow
length represent each newton of force. Label each arrow appropriately with an $F_A$, or an $F_B$.

e. What kind of motion results when two identical springs are displaced by the same amount in the same direction (e.g., when each spring is displaced to give 1.0 N of force)? How does this compare to the force of one spring displaced by twice that amount (e.g., so that it can apply 2.0 N of force)? Describe what you did and the outcome.

![Diagram of springs and forces](image)

$$F_A = \frac{1}{2} F_x$$

$$F_B = \frac{1}{2} F_x$$

Fig. 5.12.

f. Draw arrows that represents a scale drawing of the magnitudes and directions of $F_A$, $F_B$, and $F_x$.

g. In the space below, fill in the number and unit that represents each vector component.

$$F_x = \underline{\hspace{2cm}} \quad F_A = \underline{\hspace{2cm}} \quad F_B = \underline{\hspace{2cm}}$$

h. Do one-dimensional forces seem to behave like one-dimensional vector components? Why or why not?

If one dimensional forces can be described as vector components, then we can denote combinations of vectors such as those in Activity 5.6.1c and e by the equation

$$F_{x\text{ net}} = \sum F_x = F_A + F_B$$

where $F_{x\text{ net}}$ represents the sum of the vector component of two or more one-dimensional forces acting along a chosen $x$-axis. Some textbooks refer to a net force as a combined or total force. Other text authors write about the resultant force. Combined, total, resultant, or net force all refer to the same thing.
5.6.2. Activity: Calculating a Net Force

a. Let’s choose an $x$-axis that is positive to the right and negative to the left. Suppose a force $C$ has a magnitude of 1.5 N and acts toward the left on a cart and a second force $D$ has a magnitude of 0.9 N and acts toward the right on a cart. Express each force in vector component notation. **Hint:** Don’t forget to include the proper sign, the magnitude, and the units in each case.

b. Calculate the net force, $F_x^{\text{net}}$, using proper vector component notation and explain how you calculated it.

You should note that one-dimensional velocities and accelerations can also be described by vector components because they too have magnitudes and directions.

One of your goals is to continue to refine your understanding of the relationship between one-dimensional forces and motion. You just investigated how one-dimensional forces combine like vector components in terms of their ability to cause acceleration. *We can now state that acceleration caused by several forces acting in one dimension on an object that experiences very little friction is proportional to the net force on the object.*

5.7 WHAT HAPPENS WHEN THE NET FORCE IS ZERO?

Let’s consider an important special situation in more detail—the situation in which the net force acting in one dimension on an object in the absence of significant friction is zero. Start by summarizing what you already know.

5.7.1. Activity: Facts About Zero Net Force and Motion

a. Suppose we apply a force in the positive direction along an $x$-axis (toward the right) on a small low-friction cart and another force acting along the same line of equal magnitude in the negative direction on the cart (toward the left). If the law of proportionality between force and acceleration holds, what will the acceleration vector component, $a_x$, of the cart be? **Hint:** Don’t forget to include units!

\[ a_x = \text{__________} \]

b. What is the acceleration of a cart that is:
1. at rest?

\[ a_x = \text{__________} \]
2. moving with a constant negative velocity along an \( x \) axis?

\[ a_x = \underline{\text{__________}} \]

3. moving with a constant positive velocity along an \( x \) axis?

\[ a_x = \underline{\text{__________}} \]

c. Suppose an object is moving with a constant velocity and experiences no net applied forces and no friction. Is its continued motion with a constant velocity compatible with the law of proportionality between net force and acceleration or does it violate that law?

d. What can you say about the net force component on the cart mentioned in part b for each of the three cases?

We would like you to investigate whether or not a low-friction cart that has a zero net force on it can move at a constant velocity. To undertake this investigation we suggest that you apply forces in opposite directions with the same strength on a cart using two hanging weights as shown in Figure 5.13.

![Fig. 5.13. Setup showing a low-friction cart rolling on a level track as two hanging weights combine to exert zero net force on the cart.](image)

We recommend that this activity be done as a demonstration with the entire class participating. The items needed for the demonstration include:

- 1 high table
- 1 smooth ramp or level surface 2 meters long
- 1 small low friction dynamics cart
- 2 lengths of string, 2 m
- 2 low friction pulleys
- 2 mass pans, 1 kg
- 1 set of slotted masses (5 g to 100 g)
- 1 computer-based laboratory system
Before making the observations you should level the track and balance the masses on either side with the smaller slotted masses so that, if the cart is stationary at first, it stays at rest and doesn’t accelerate in either direction. An electronic force sensor should be firmly attached to the cart to help verify that the pulling forces on the cart don’t change as the cart moves to the right or the left during the observations.

### 5.7.2. Activity: Motion with No Net Force

**a.** Suppose a cart receives a brief push that starts it moving in one direction or another. If there is no net force on it after the push, what do you predict its motion will be like? Try to imagine that there is no friction acting on the cart. Explain the reasons for your prediction.

**b.** Is your prediction compatible with the proportional relationship between force and acceleration that you discovered previously?

**c.** Observe what happens after the cart is pushed in one direction and allowed to move freely with no net force. Describe your observation. Is the velocity of the cart constant or decreasing. Does the cart seem to be accelerating?

**d.** The observation you just made should enable you to state Newton’s First Law of Motion. Please finish the statement in a way that is compatible with your actual observation.

**NEWTON’S FIRST LAW:** If an object moving at a constant velocity, \( \vec{v} \), without friction experiences no net force, it will . . .
Remarks About Newton’s First Law

Newton’s First Law is of deep significance because it allows an observer who is moving at a constant velocity with respect to another observer to discover the same laws of motion. For example, suppose you observe that the small cart is at rest in the laboratory and your partner makes the same observation while moving away from you at a constant velocity on a Kinesthetic cart. Your partner will see the small cart moving away with a constant velocity. But both of you can agree that the cart is not accelerating and therefore is experiencing no net force!

RELATING FORCE, MASS, AND ACCELERATION

5.8 DEFINING AND MEASURING MASS

A good friend calls you in a panic. His battery is dead and he needs to have you come outside and help push his car to get it started. He needs to have you get it moving at about 12 mph so he can throw it in gear and turn over the engine. You blithely answer sure, you’ll be right out. Then you remember that your friend owns two vehicles—a large delivery van and a smaller sports car. Since you can exert only so much force and you’re feeling like an 80 lb weakling, you hope that your friend is driving the easiest of the two cars to push.

5.8.1. Activity: Causing a Car to Accelerate

a. Which vehicle do you think would be easier to accelerate from rest to a speed of 12 mph with a given force—a small car or a large van? Explain.

b. What characteristic of an object seems to determine how much the object accelerates for a given force?

Somehow the magnitude of force required to cause an object to accelerate by a given amount is related to the “amount of stuff” being accelerated. It is pretty obvious to most people that if there is more stuff, then more force will be required to accelerate it. But suppose we double the amount of stuff. Will that mean that twice the force is needed to accelerate double the stuff?

The stuff we are referring to is what scientists usually call mass. Let’s take some time to consider the question of what mass is and how we might measure it.
What Is Mass?
Philosophers of science are known to have great debates about the definition of mass. If we assume that mass refers somehow to “amount of stuff,” then we can develop an operational definition of mass for matter that is made up of particles that appear to be identical. We can assume that mass adds up and that two identical particles when combined have twice the mass of one particle; three particles have three times the mass; and so on. But suppose we have two objects that have different shapes and are made of different stuff, such as a small lead pellet and a silver coin. How can we tell if these two entities have the same mass?

5.8.2. Activity: Ideas About Mass and Its Measurement

a. Attempt to define mass in your own words without using the word “stuff.”

b. How many different ways can you think of to determine whether a lead pellet and a silver coin have the same mass?

c. Suppose you find that the lead pellet and the silver coin seem to have the same mass. How could you create “stuff” that has twice the mass of either of the original objects?

Using a Mass Balance
One time-honored way that people have used to compare the mass of two objects is to put them on a balance. If they happen to balance each other, we say that the “force of gravity” or the force of attraction exerted on them by the earth is the same, so they must have the same mass.
Fig. 5.15. A common method of determining mass that assumes two objects have the same if they experience the same gravitational force.

Actually, if we balance gravitational forces as the method of determining mass, we are only determining a *passive gravitational mass*. A passive gravitational mass is proportional to the force of attraction exerted by the earth on the mass. How can we use a balance to measure the mass of any object relative to a standard mass?

Let’s do a thought experiment. Suppose Maya makes the outrageous claim that her dove has the same mass as 79 silver quarters and is worth her weight in quarters. Can you doublecheck her claim using a balance, a quarter as the “standard mass,” and a pile of sand? **Hint:** This is an exercise in basic logic.

5.8.3. Activity: Using a Balance to Measure an Arbitrary Mass

Explain how you might measure the passive gravitational mass of Maya’s dove using the balance, sand, and standard coin.

**Other Ways to Measure Passive Gravitational Mass**

In most modern laboratories, spring scales and electronic scales that are easier to use now replace the old-fashioned balance. As the earth attracts a mass hanging from a spring, the spring will stretch. A mass placed on the platform of an electronic scale will cause it to depress. The amount of depression can be detected electronically.
5.9 HOW MASS AFFECTS MOTION

You have already verified a law of proportionality between one-dimensional force and acceleration when little friction is present. This law can be expressed in the form

\[ F_{\text{net}} = ma_x \]  

(5.1)

where \( F_{\text{net}} \) is the net force exerted on an object, \( m \) is the slope of the graph of \( F_{\text{net}} \) vs. \( a_x \) or the constant of proportionality, and \( a_x \) is the acceleration along the line that the net force acts. We know that this constant of proportionality, \( m \), which represents a resistance of an object to acceleration, doesn’t necessarily have anything to do with gravity.

Since it requires more force to accelerate more gravitational mass our intuition tells us that the proportionality constant ought to be related to passive gravitational mass. Is it possible that the passive gravitational mass of an object is the same as the proportionality constant, or slope, relating \( F \) and \( a \)? In other words, will accelerating twice as much passive gravitational mass by the same amount take twice as much force? This is the question we posed in Section 5.7.

To answer this question you should investigate the forces and accelerations that arise when you push and pull on rolling carts having different passive gravitational masses. This investigation is basically an extension of the one you undertook in Section 5.5. For the next activity you will need:

- 1 balance
- 2 pieces of string (to use with the balance)
- 1 electronic scale
- 1 small low-friction dynamics cart
- 1 set of assorted masses (1 g, 2 g, 5 g, 10 g, 20 g, 50 g, 100 g, 200 g)
- 1 smooth ramp or level surface 1–3 meters long
- 1 computer-based laboratory system
- 1 motion software
- 1 ultrasonic motion sensor
- 1 force sensor
- 1 adapter bracket (to attach a force sensor to the cart)
- 1 spring scale, 10 N (to calibrate the force sensor if needed)

| Recommended Group Size: | 2 | Interactive Demo OK?: | Y |

Before doing the activity, attach a force sensor with a hook on its end firmly to the cart. Place the cart on a smooth level track or surface.

![Fig. 5.17. Setup showing a motion sensor tracking the acceleration of a cart rolling on a level track as a force sensor detects the pushes and pulls on it. As usual, the cart must be at least 0.5 m away from the motion detector at all times.](image-url)

\( ^* \) Because of its linearity, low noise, and built-in calibration, we recommend that the PASCO Force Sensor (models CI-6537 or CI-6618) be used in this activity.
When you did a similar observation in Section 5.5 you collected data for force vs. time and acceleration vs. time to see the shapes of the graphs. Since you were also interested in the relationship between force and acceleration you plotted a graph of this relationship using some sample data and found the slope of the graph. This time you should set up the motion software to display three graphs:

1. force vs. time
2. acceleration vs. time
3. force vs. acceleration

If needed, calibrate the force sensor to read forces between $-5$ and $+5$ N.

**5.9.1. Activity: Measuring Acceleration vs. Force**

a. Zero the force sensor (if needed) and record data as you grip the force sensor hook firmly to push and pull the cart back and forth on the track smoothly. Sketch the force and acceleration graphs you observed. **Note:** If your acceleration graph seems rough and has spikes on it, (1) you should set the averaging for the velocity and acceleration data at something between 5 and 15 points so that small uncertainties in data do not appear on the acceleration graphs, and (2) you should try to push and pull with a maximum force of about 4 N.

![Graphs](image)

b. Use the linear fitting feature in the motion software to find the slope of the graph of force vs. acceleration. What is the value of the slope and its standard error? Be sure to include units.

\[ \text{Slope}: m = \]

\[ \text{Standard Deviation from the Mean}: \quad \text{SDM} = \]

---

**Doubling the Gravitational Mass**

Suppose you put enough mass on the cart to double its mass. How much more force must you exert to push and pull on the more massive cart so that its acceleration vs. time graph looks about the same as the graph you obtained in Activity 5.9.1a?
5.9.2. Activity: Acceleration vs. Force for Twice the Mass

a. Suppose you were able to push and pull on a cart having twice the mass as before so that the acceleration vs. time is approximately the same. Do you think the slope of the $F$ vs. $a$ graph will increase, decrease, or remain the same? Explain.

b. Sketch the predicted shapes of all three graphs.

c. Use the balance to compare the combined mass of the cart, force sensor, and sensor holder to that of a collection of assorted masses. Include enough assorted masses to just balance the gravitational mass of the cart-sensor system.

Place this combination of masses on the cart so that the cart with added masses, force sensor, and sensor holder now have twice the gravitational mass as before. Zero the force sensor (if needed) and record data as you grip the force sensor hook firmly to push and pull the cart back and forth on the track *smoothly*. Sketch the force and acceleration graphs you observed.
d. Using the linear fit feature in the motion software, indicate the value of the slope and the standard error of the force vs. acceleration graph. Include units for the slope.

Slope: \( m = \)

Standard Deviation from the Mean: \( \text{SDM} = \)

e. Compare the slope you found here with that found in Activity 5.9.1. Does gravitational mass seem to be the constant of proportionality between force and acceleration?

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**Newton’s Second Law**

The slope of the linear graph relating the one-dimensional net force and the acceleration caused by that force is defined as the *inertial mass*. By defining inertial mass in this way, we have developed sensible definitions of force and mass that lead to Newton’s Second Law of Motion for one-dimensional motions in the absence of friction. Newton’s Second Law expressed in vector component notation for an object having a total inertial mass of \( m \) is simply

\[
F_{x,\text{net}} = \sum F_x = ma_x
\]  

(5.2)

where \( F_{x,\text{net}} \) is a vector component representing the net one-dimensional force on the object, and \( a_x \) is a vector component representing the acceleration caused by the net force.

**Inertial and Passive Gravitational Mass**

In the last activity you have shown that within the limits of experimental uncertainty inertial and passive gravitational masses are the same. This makes the definition of inertial mass as the proportionality constant between acceleration and force seem less arbitrary.
It is not obvious that these two definitions of mass—passive gravitational and inertial—should yield exactly the same results. This equivalence is assumed in both Newton’s theory of gravity and Einstein’s general relativistic modifications of it. In fact, sophisticated experiments have shown that within the limits of experimental uncertainty, there is no difference between the two types of mass to within one part in 1011.

**Standard SI Units for Mass and Force**

As you already know, the Systeme Internationale, or SI, system of units provides us with a standard set of units that we often use. This system was established in 1960 to provide units that all scientists throughout the world should use. The SI units for fundamental quantities used in mechanics, including mass, are shown below.

**SI UNITS FOR MECHANICS**

*Length:* A **meter** (m) is the distance traveled by light in a vacuum during a time of $1/299,792,458$ second.

*Time:* A **second** (s) is defined as the time required for a light wave given off by a cesium–133 atom to undergo $9,192,631,770$ vibrations.

*Mass:* A **kilogram** (kg) is defined as the mass of a platinum–iridium alloy cylinder kept in a special chamber at the International Bureau of Weights and Measures in Sévres, France.

The electronic balance and spring scales often used in laboratories have been calibrated using replicas of the “real” standard kilogram mass kept in a vault in France. These fundamental units and Newton’s Second Law can also be used to define the newton as a unit of force. The newton is defined in terms of mass and acceleration as shown in the box below.

---

**The Force Unit Expressed in Terms of Length, Mass and Time** Force: A **newton** (N) is defined as that force, which, when acting on a 1-kg mass, causes an acceleration of 1 m/s/s.

---

**Do the Standard Units Work Together?**

With the exception of a device called an inertial balance, essentially all the common equipment used in laboratories today measures passive gravitational mass rather than inertial mass. Thus, if you determine the mass in kilograms of your cart and force sensor system using an electronic balance, you can compare it to the inertial mass you found by accelerating your cart system in Activity 5.9.1.

Since you measured forces in newtons and accelerations in m/s/s, the passive gravitational mass readings from a well-calibrated electronic balance and the inertial mass readings from the slopes of your $F$ vs. $a$ lines should be the same. Are they?

---

### 5.9.3 Activity: Gravitational vs. Inertial Masses in Kilograms

- Use the electronic balance to determine the passive gravitational mass of your cart with the force sensor attached to it.
\[ m_{\text{grav}} = \text{kg} \]

**b.** What is the inertial mass of your cart with the force sensor attached to it as reported in Activity 5.9.1?

\[ m_{\text{inertial}} = \text{kg} \]

**c.** Are they the same within the limits of experimental uncertainty?

**d.** What do you think are the possible sources of systematic error and experimental uncertainty in your two measurements?

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### 5.10 Summarizing Newton’s First and Second Laws

The main purpose of Unit 5 has been to explore the relationships between forces on an object, its mass, and its acceleration. You have been trying to develop Newton’s first two laws of motion for one-dimensional situations in which all forces lie in a positive or negative direction along the same line and in which there is very little friction present.

**5.10.1. Activity: Newton’s Laws in Your Own Words**

Express Newton’s Laws in your own words clearly and precisely.

- The First Law (the one about constant velocity):

- The Second Law (the one relating force, mass, and acceleration):
5.10.2. Activity: Newton’s Laws in Equation Form

Express Newton’s Laws in equations in terms of the acceleration or velocity vector component, the net force on an object, and its mass:

The First Law: If $F_{\text{net}}^x = \sum F_x = 0$ then $v_x = \ldots$ or

**Note:** The use of the equal sign does not signify that an acceleration is the same as or equivalent to a force divided by a mass, but instead it spells out a procedure for calculating the magnitude and direction of the acceleration of a mass while it is experiencing a net force. What we assume when we believe in Newton’s Second Law is that a net force on a mass causes an acceleration of that mass.

The Second Law: If $F_{\text{net}}^x \neq 0$ then $a_x =$
Final Comments on Force, Mass, and Motion

You started your study of Newtonian dynamics in this unit by attempting to develop the concept of force. Initially, when asked to define force, most people think of a force as an obvious push or pull such as a punch to the jaw or the tug of a rubber band. By studying the acceleration that results from a force when little friction is present, we can come up with a second definition of force as that which causes acceleration. These two alternative definitions of force do not seem to be the same at all. Pulling on a hook attached to a wall doesn’t seem to cause the wall to move. An object dropped close to the surface of the earth accelerates and yet there is no visible push or pull on it.

The genius of Newton was to recognize that he could define net force as that which causes acceleration. He reasoned that if the applied forces did not account for the degree of acceleration then other “invisible” forces must be present. A prime example of an invisible force is that of gravity—the attraction of the earth for objects.

Finding invisible forces is hard sometimes because some of them are known as passive forces because they only seem to act in response to either the motion of an object or other forces on it. Friction forces are one example of passive forces. They are not only invisible, but they only crop up during motions for the purpose of inhibiting the motion. The passive nature of friction is obvious when you think of a person riding on a garbage bag and sliding along the floor at constant velocity under the influence of an applied force.

According the Newton’s First Law, a person moving at a constant velocity must have no net force on her. Newton thought that the applied force in one direction had to be opposed by a friction force acting in the other direction to oppose her motion. The friction force must be passive because, if the applied force is discontinued, the friction force does cause the sliding person to slow down until she has no motion. Then the friction force stops acting. If it didn’t stop acting, the person would slow down and then turn around and speed up in the opposite direction. This doesn’t happen!

During the rest of our study of the Newtonian formulation of classical mechanics your task will be to discover and invent new types of active and passive forces so that you can continue to explain and predict motions using Newton’s Laws. In the next unit you will be using Newton’s Laws to explain why sliding masses that have no visible forces on them slow down and come to rest, and to learn why masses fall when dropped close to the surface of the Earth. You will also learn how to extend Newton’s Laws to two-dimensional situations when the motion of an object and the forces that act on it do not lie along the same line.